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Course Handout

Analog Communications

Specialty: 3rd year in Telecommunications

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Preface

This document has been designed for third-year Telecommunications Engineering students to provide a comprehensive understanding of the fundamental principles of radio frequency (RF) communication systems and their practical applications in modern analog transmission. The content aims to build a solid foundation in RF concepts, modulation techniques, and essential electronic components used in communication systems.

The course begins with the basics of radio frequency, covering essential notions such as the structure of transmitters and receivers, electromagnetic waves, bandwidth, and logarithmic units expressed in decibels. These introductory concepts form the groundwork for understanding signal behavior in the RF domain.

The second chapter, Basic Functions in Analog Transmission, introduces key analog building blocks such as filters, oscillators, and analog multipliers. Students will study different types of filters and their frequency responses, analyze oscillator design based on the Barkhausen criterion, and explore the practical role of analog multipliers in modulation and signal processing.

The subsequent chapters address the core principles of modulation techniques used in analog communication.

Chapter 3 focuses on Amplitude Modulation (AM), including Double Sideband (DSB), Single Sideband (SSB), and conventional AM, along with their corresponding demodulation techniques.

Chapter 4 presents Angle Modulation, which encompasses Frequency Modulation (FM) and Phase Modulation (PM). The chapter explores their mathematical representation, spectral characteristics, and implementation of modulators and demodulators.

Further sections deal with the effect of noise on analog communication systems, highlighting the influence of random disturbances on both linear and angle modulation schemes. The Superheterodyne Receiver is then introduced as a cornerstone of practical RF communication, explaining its role in frequency conversion and signal amplification.

Finally, the module concludes with an in-depth study of the Phase-Locked Loop (PLL)—a fundamental control system widely used in synchronization, demodulation, and frequency synthesis applications.

Overall, this course is intended to combine theoretical analysis with practical understanding, preparing students to analyze, design, and optimize analog communication systems. Through the study of these chapters, students will gain the knowledge required to bridge the gap between basic electronic circuits and advanced communication systems used in current telecommunication technologies.

Dr. HAlASSI ABDEREZZAQ

Contents

1. Basics of radio frequency

1.1 Introduction

The study of radio frequency (RF) fundamentals forms the cornerstone of all wireless communication systems. This chapter introduces the basic principles that govern the generation, transmission, and reception of radio signals.

We begin by exploring the architecture of a communication system, highlighting the main functional blocks of both the transmitter and the receiver. Understanding how information is modulated, transmitted through a channel, and finally recovered is essential for grasping more advanced communication techniques.

Next, the chapter reviews the basic concepts of radio transmission, including the nature of electromagnetic waves, their propagation characteristics, and the role of bandwidth in determining system performance. The section on logarithmic units and decibel representation provides the mathematical tools needed to express power ratios and signal levels efficiently, a key skill in RF engineering.

The final section of this chapter introduces the concept of passband (or narrowband) signals, which are central to RF communication. Through the study of the Hilbert transform, analytic signal, and complex envelope, students learn how to represent and manipulate signals in both time and frequency domains—an essential step before tackling modulation and demodulation techniques.

1.2 Communication system architecture

The general schematic of a radio communication system is shown in Figure 1.1. This architecture is the same for most radio communication systems, regardless of the type of information transmitted, whether analog or digital.

1.2.1 Transmitter structure

The transmitter consists of,

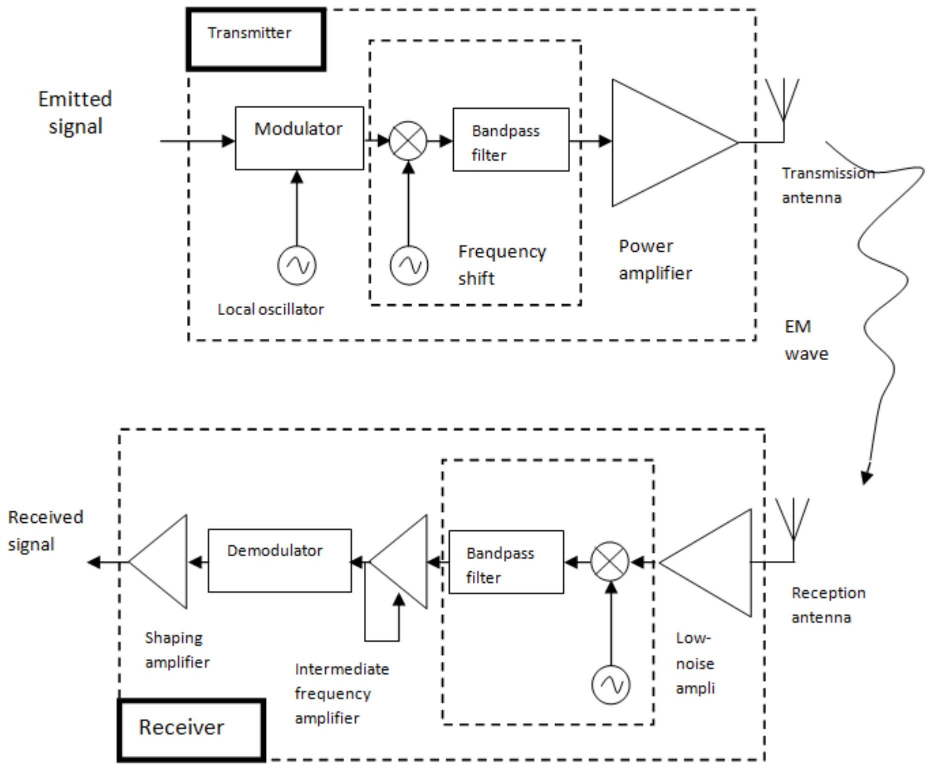


Figure 1.1: Functional diagram of a radio communication system

Modulator

The modulator has two inputs: one input for the information message (modulating signal) and another input for the local oscillator, typically a carrier signal.

Oscillator

It is the device that generates the carrier signal. Its frequency must be stable and precise; it should not vary over time or under the influence of external parameters (temperature, power supply voltage, etc.).

Frequency shift

This stage is not always present. If the emission frequency is low (below GHz), the modulation is performed directly at the emission frequency, and the output of the modulator drives the power amplifier. If the emission frequency is high, the modulation is done at a frequency on the order of MHz, known as the intermediate frequency.

Power amplifier

Its role is to deliver the maximum power to the transmission antenna. Its most important characteristics are the maximum emission power and its linearity (it should not distort the output signal).

1.2.2 Receiver structure

The receiver is composed of:

Low-noise amplifier

It is a selective amplifier; it should only amplify the band of the received signal and eliminate all other frequencies. It should introduce as little noise as possible.

Frequency shifting

This stage brings the received signal down to a lower frequency called the "intermediate frequency," at which the demodulator circuit operates. The bandpass filter serves to retain only the useful part of the received signal.

AGC (Automatic Gain Control) amplifier

This high-gain amplifier helps recover the dB lost during propagation. It allows for operation at a constant level to drive the demodulator.

Demodulator

Allows for the recovery of the transmitted signal from the modulated signal. It operates at the intermediate frequency.

Shaping circuit

It provides amplification and/or shaping of the demodulated signal, and its structure depends on the nature of the message to be transmitted.

1.3 Basic concepts of radio transmission

1.3.1 Electromagnetic waves

In a radio link, it is the electromagnetic wave that carries the information to be transmitted. It consists of an electric field $\vec{E}$ and a magnetic field $\vec{H}$ that are perpendicular to each other. In a vacuum, the wave propagates at the speed of light 3.10^8 m/s and at a frequency given by:

$$f = \frac{c}{\lambda} \quad Hz \tag{1.1}$$

With λ (in m), the wavelength defined as the distance between two consecutive maxima, but also the distance traveled by the wave during the duration of a period.

1.3.2 bandwidth

The bandwidth is defined as the range of frequencies in which the signals applied to the input of the transmission medium do not suffer excessive attenuation and have an output power above a given threshold after passing through the medium. Two quantities are defined:

- Bandwidth of a transmission medium
- Spectral occupancy of a signal (signal bandwidth)
  For effective transmission, it must be ensured that:
- The spectral occupancy of the signal is less than the bandwidth of the medium,
- The spectrum of the signal coincides with that of the bandwidth

1.3.3 Logarithmic units (decibels)

The decibel is a unit originally introduced in acoustics to mathematically express the fact that auditory sensation is proportional to the logarithm of acoustic power. The main reasons for using a logarithmic scale are:

- The significant variation in power proportions (for example, from W to nW, which becomes more manageable in logarithmic units).
- The representation of the frequency response of a linear system has straight asymptotes in logarithmic units.
- The use of logarithmic values allows for the simple addition of gains and attenuations encountered in a transmission chain during an operation called link budget.

1.3.4 Power ratio in dB

If P_1 and P_2 are two powers, input and output respectively, the gain in decibels $\mathbf{G_{dB}}$ of a transmission element is defined as follows:

$$\mathbf{G_{dB}} = 10\, log \frac{P_2}{P_1} \quad (\mathbf{dB}) \tag{1.2}$$

The attenuation (or loss) of the signal will be defined in the same way as the gain but will have a negative value. A metal cable transmitting a signal while reducing its power level by a factor of 20 results in an attenuation of -13 **dB**.

1.3.5 Relative power

It is defined with respect to a reference value $P_{\mathbf{ref}}$ and is expressed as follows:

$$\mathbf{P_{dB,relative}} = 10\, log \frac{P}{P_{\mathbf{ref}}} \tag{1.3}$$

For a reference power of $P_{\mathbf{ref}} = 1$ **W**, the relative power will be expressed in **dBW**. If the power $P_{\mathbf{ref}} = 1$ **mW**, the relative power is expressed in **dBm**. An input power of 500 μ**W** corresponds to a relative power of -3 **dBm**. We define:

$$0\ \mathbf{dBW} = 30\ \mathbf{dBm} \qquad\qquad 0\ \mathbf{dBm} = -30\ \mathbf{dBW}$$

1.4 Passband signals (narrowband signals)

In this section, we examine the characteristics in both the time and frequency domains of a class of signals commonly encountered in communication system analysis. This class of signals is known as passband signals (or narrowband signals). The concept of passband signals is a generalization of monochromatic signals, and our analysis of the properties of these signals is based on that used for monochromatic signals.

1.4.1 Hilbert transform

We know that a phase shift of $\frac{\pi}{2}$ of the signal $\cos 2\pi f_0 t$ leads to the signal $\sin 2\pi f_0 t$. The Hilbert transform is the linear filtering operation that represents this $\frac{\pi}{2}$ phase shift for all spectral components of any given signal. The Hilbert transform of the centered real signal $x(t)$, is denoted by $\hat{x}(t)$ and is defined as follows,

$$\hat{x}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(u)}{t-u}\, du \tag{1.4}$$

From the perspective of signal theory, we observe that this integral can be interpreted as a convolution product. We can write,

$$\hat{x}(t) = x(t) * \mathbf{Dis}(\frac{1}{\pi t}) \tag{1.5}$$

with, **Dis(.)** denotes the distribution. We deduce that the operation that transforms the signal $x(t)$ into its Hilbert transform $\hat{x}(t)$ is a linear filtering operation whose impulse response, denoted by $h(t)$, is given by,

$$\mathbf{h(t)} = \mathbf{Dis}(\frac{1}{\pi t}) \tag{1.6}$$

and whose transfer function $\mathbf{H}(f)$, defined as the Fourier transform of $\mathbf{h}(t)$, is given by,

$$\mathbf{H}(f) = \mathbf{FT}(\mathbf{h}(t)) = -\mathbf{j}\ \mathbf{sign}(f) \tag{1.7}$$

where $\mathbf{sign}(f)$ denotes the sign function defined as follows,

$$\mathbf{sign}(f) = \begin{cases} 1 & \mathbf{si}\ \ f > 0 \\ 0 & \mathbf{si}\ \ f = 0 \\ -1 & \mathbf{si}\ \ f < 0 \end{cases} \tag{1.8}$$

The resulting filter is called a Hilbert filter. It is a pure phase shifter, as its frequency gain is equal to 1 for all f, and it introduces a phase shift of $\frac{\pi}{2}$ for negative frequencies and $-\frac{\pi}{2}$ for positive frequencies.

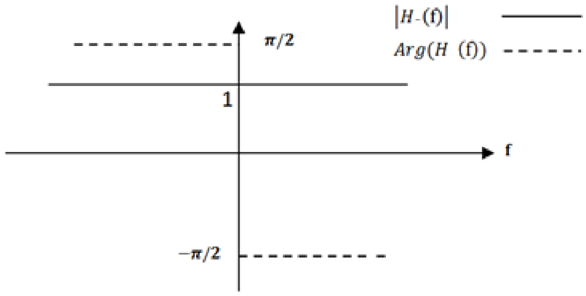


Figure 1.2: Transfer function of the Hilbert filter

It is well known that the Hilbert transform of the cosine signal is the sine signal, which is also the reason the Hilbert filter is referred to as a quadrature filter. Indeed, the signal $x(t) = \cos 2\pi f_0 t$ has the following Fourier transform.

$$\mathbf{X}(f) = \frac{1}{2}(\delta(f - f_0) + \delta(f + f_0)) \tag{1.9}$$

$$\hat{\mathbf{X}}(f) = -j\mathbf{sign}(f)\mathbf{X}(f) = \frac{1}{2j}(\delta(f - f_0) - \delta(f + f_0)) \tag{1.10}$$

Using the inverse Fourier transform, we obtain,

$$\hat{\mathbf{x}}(t) = \sin 2\pi f_0 t \tag{1.11}$$

t can be noted that the Hilbert filter is non-causal and thus physically unrealizable. However, it is possible to create a finite-delay approximation over a limited frequency band.

1.4.2 Analytic signal

The analytic signal is a complex representation of a real signal. It is a well-known and widely used representation, as it allows us to represent the real signal $\cos 2\pi f_0 t$ by the complex signal $\exp(j2\pi f_0 t)$ the concept of the analytic signal generalizes this particular case. We know that the Fourier transform of a real signal has the property of Hermitian symmetry (magnitude and real part are even, while the argument and imaginary part are odd). Consequently, to fully define a real signal in the frequency domain, it is not necessary to know its Fourier transform for all frequency values; it suffices to know it only over R^+. We can thus, without loss of information, represent a real signal $x(t)$ by a signal $z_x(t)$ whose Fourier transform $Z_x(f)$ coincides (up to a multiplicative factor introduced for convenience) with the Fourier transform $X(f)$ of the real signal $x(t)$ for positive frequencies, and is zero for negative frequencies, as follows:

$$Z_x(f) = \begin{cases} 2X(f) & \textbf{if } f > 0 \\ 0 & \textbf{if } f < 0 \end{cases} \tag{1.12}$$

The signal $z_x(t)$ is called the analytic signal associated with the real signal $x(t)$. It is necessarily a complex signal since, by its very construction, it does not possess the property of Hermitian symmetry. In general, a signal $z(t)$ is said to be analytic if its spectrum $Z(f)$ is zero for negative frequencies.

The previous relationship describes the analytic signal in the frequency domain, and we can easily deduce its temporal expression $z_x(t)$. Indeed, we can write,

$$Z_x(f) = 2U(f)X(f) \tag{1.13}$$

where $U(f)$ denotes the unit step function in frequency.

The analytic signal associated with $x(t)$ is thus obtained by applying $x(t)$ to the input of a linear filter with a transfer function $H(f) = 2U(f)$, known as the analytic filter. Using the inverse Fourier transform, we obtain,

$$u(t) = \frac{1}{2}(\delta(t) + j\mathbf{Dis}(\frac{1}{\pi t})) \tag{1.14}$$

we can write,

$$z_x(t) = 2u(t) * x(t) = (\delta(t) + j\mathbf{Dis}(1/\pi t)) * x(t) \tag{1.15}$$

Let us consider, taking into account the results from the previous paragraph.

$$z_x(t) = x(t) + j\hat{x}(t) \tag{1.16}$$

In particular, from this general definition, we find that the analytic signal associated with the signal $\cos(2\pi f_0 t)$ is the signal $\exp(2\pi f_0 t)$. The analytic signal $z_x(t)$ associated with $x(t)$ is thus a complex signal whose real and imaginary parts are Hilbert transforms of each other.

1.4.3 Narrowband signal

A real signal is said to be narrowband if its Fourier transform is zero (or practically negligible) outside of two frequency bands defined by $f_1 < |f| < f_2$. The spectrum of such a signal is illustrated in the figure. The central frequency f_0 is often referred to as the carrier frequency.

To characterize these signals, it is possible to use a representation that abstracts the position of the spectrum on the frequency axis, meaning the value f_0 this is the purpose of the representation by the complex envelope. This representation allows us to replace a narrowband real signal with a complex signal in general, but one whose spectrum is located in the low frequencies, and it proves very useful for analytical calculations.

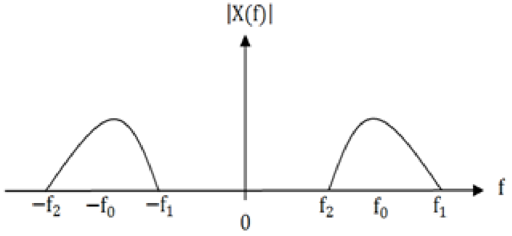


Figure 1.3: Spectrum of a narrowband signal

1.4.4 Complex envelope

The complex envelope of the real narrowband signal $x(t)$ is defined as the signal, denoted $e_x(t)$, whose Fourier transform $E_x(f)$ is obtained from the associated analytic signal $z_x(t)$ by a translation of $(-f_0)$ on the frequency axis,

$$E_x(f) = Z_x(f + f_0) \tag{1.17}$$

The complex envelope of a signal $x(t)$ thus has a spectrum of the same shape (in particular, the same bandwidth) as that of the signal itself, but centered around zero frequency. Its temporal representation is obtained by the inverse Fourier transform of relation (1.17), which gives

$$e_x(t) = z_x(t)\exp(-j2\pi f_0 t) \tag{1.18}$$

$$e_x(t) = (x(t) + j\hat{x}(t))\exp(-j2\pi f_0 t) \tag{1.19}$$

Conversely, we can express a narrowband signal from its complex envelope using the relation,

$$x(t) = \Re[z_x(t)] = \Re[e_x(t)\exp(j2\pi f_0 t)] \tag{1.20}$$

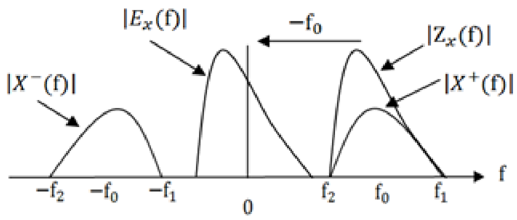


Figure 1.4: Spectra: of the analytic signal and the complex envelope of the signal

According to the decomposition of $e_x(t)$ either into real and imaginary parts, or into magnitude and argument, we can derive two representations of narrowband signals,

- Decomposition in phase and quadrature.
- Decomposition in magnitude and argument.

1.5 Conclusion

In this chapter, we have developed a foundational understanding of radio frequency principles and their role in communication systems. By examining the structure of transmitters and receivers, we gained insight into the signal flow from source to destination.

We also studied key physical and mathematical concepts—electromagnetic wave propagation, bandwidth, and logarithmic units—which are indispensable for quantifying and analyzing RF signals. Finally, by introducing narrowband signal representation through the Hilbert transform and complex envelope, we prepared the groundwork for more advanced topics such as amplitude and angle modulation.

These concepts together provide the essential background for analyzing, designing, and optimizing analog and digital communication systems. The understanding of RF basics developed here will serve as a bridge toward practical applications in modulation, filtering, and receiver architecture discussed in the following chapters.

2. Basic functions in analog transmission

2.1 Introduction

Analog communication systems rely on several fundamental electronic functions that allow the generation, transmission, and processing of continuous-time signals. This chapter focuses on three essential building blocks used in analog transmission: filters, oscillators, and analog multipliers.

The first section introduces analog filters, which are crucial for selecting desired frequency components and rejecting unwanted ones. Students will study the basic concepts of filtering, explore different types such as low-pass, high-pass, band-pass, and band-stop filters, and analyze both first-order and second-order active filters. These filters form the foundation for many analog and RF circuit designs used in transmitters and receivers.

The second part of the chapter deals with oscillators, which are circuits capable of generating periodic signals without an external input. The basic theory of oscillation and the Barkhausen criterion are discussed to explain the conditions required for sustained oscillation. Practical oscillator configurations such as the phase-shift, Wien bridge, Hartley, and Colpitts oscillators are then presented, demonstrating their operation using both transistors and operational amplifiers.

Finally, the chapter concludes with the study of analog multipliers, devices that perform the multiplication of two analog signals. After reviewing their basic characteristics, their applications in modulation, frequency conversion, and automatic gain control are explored.

Through these three sections, this chapter builds the essential knowledge needed to understand how analog systems shape, generate, and manipulate signals for communication purposes.

2.2 Analog filter

An electronic system is always designed to work in a well defined and necessarily limited frequency range. Indeed, the properties of electronic components are always defined over a frequency range.

Table 2.1: Frequency Band Name

Frequency Band Name	Frequency Range
Very Low Frequency (VLF)	< 30 kHz
Low Frequency (LF)	$30 - 300$ kHz
Medium Frequency (MF)	$0.3 - 3$ MHz
High Frequency (HF)	$3 - 30$ MHz
Very High Frequency (VHF)	$30 - 300$ MHz
Ultra High Frequency (UHF)	$0.3 - 3$ GHz
Super High Frequency (SHF)	$3 - 30$ GHz
Extremely High Frequency (EHF)	$30 - 300$ GHz

2.2.1 Basics

Frequency Bands

BODE diagram

is the most commonly used method for the analysis and design of linear feedback control systems. There are two BODE diagrams for the transfer function, one is the magnitude versus frequency plot and the other is phase angle versus frequency plot. The frequency is taken in the x axis and drawn in a semi-log graph. Inthe magnitude versus frequency plot, decibel denoted by dB is plotted instead of magnitude. The magnitude of $|H(jw)|$ is expressed as $20log_{10}|H(jw)|$

2.2.2 Types of filters

There are four commonly used types of filters as shown in figure 2.1

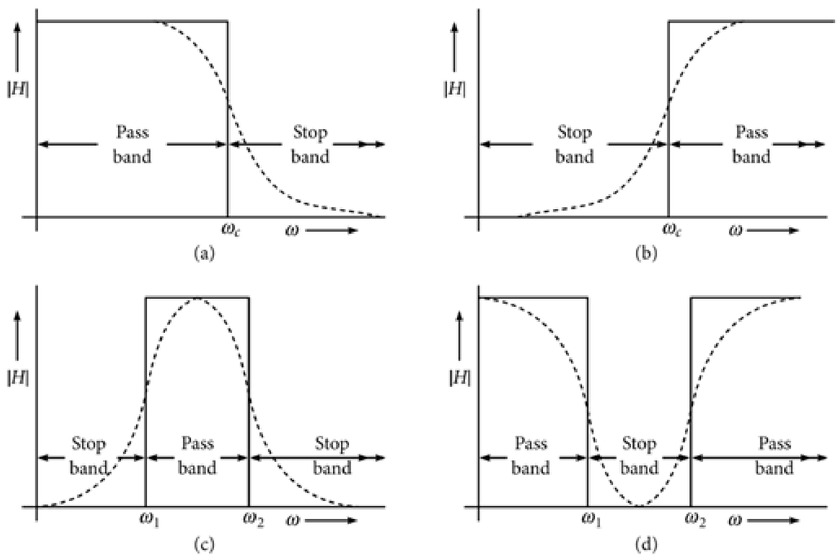


Figure 2.1: Firm lines show ideal response and dashed lines shows practical response for (a) low pass, (b) high pass, (c) band pass, and (d) band stop filters.

Low Pass Filters when the frequency range of the pass band is from $w = 0$ to $w = w_c$ and the stop band extends from $w = w_c$ to ∞, the filter is known as a Low Pass Filter (LPF) as shown in Fig

2.1.a. here w_c is called the cut-off frequency.
High Pass Filters When the frequency range of the pass band is from $w = wc$ to $w = \infty$ and the stop band extends from $w = 0$ to w_c, the filter is known as a High Pass Filter (LPF) as shown in Fig 2.1.b.

2.2.3 First order active filters

To decrease the size of electronic filters in accordance with modern technology, replacement of inductors with appropriate elements becomes essential. Utilization of active devices, analog with resistance and capacitance elements, with simulated inductance, led to active filters.

First order Low Pass filter

The transfer function of a physically realizable filter using a finite number of elements has to be real rational function. The rational function is a ration of polynomials in the complex frequency jw.

$$H(jX) = \frac{H_0}{1 + jX} \tag{2.1}$$

where, $X = \frac{\omega}{\omega_c} = \frac{f}{f_c}$.
H_0, is the maximum amplification.

Exercise 2.1 Find the transfer function for the circuit shown in Figure 2.2, ■

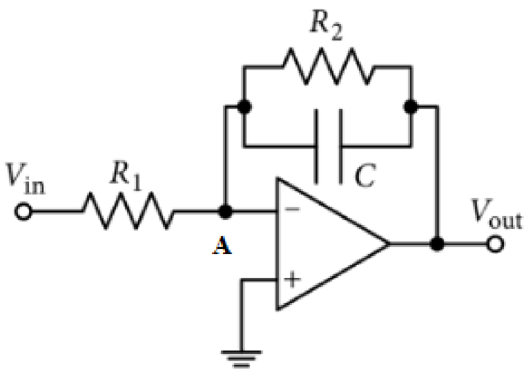


Figure 2.2: First order Low pass filter

Solution 2.1

We assume that the operational amplifier (OA) is ideal $V^+ = V^-$.
By applying Millman's theorem to point **A**,

$$V_A = \frac{\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} + jC\omega V_{out}}{\frac{1}{R_1} + \frac{1}{R_2} + jC\omega} = 0 \Rightarrow \left(\frac{1}{R_2} + jC\omega\right) V_{out} = -\frac{V_{in}}{R_1}$$

$$H(j\omega) = \frac{V_{out}}{V_{in}} = -\frac{1}{R_1}\frac{1}{\frac{1}{R_2} + jC\omega} \Rightarrow \quad H(j\omega) = -\frac{R_2}{R_1}\frac{1}{1 + jR_2C\omega}$$

By identification with equation 2.1 we obtain, $H_0 = -\frac{R_2}{R_1}$ and $\omega_c = \frac{1}{CR_2}$.

Bode diagram

We assume that the $R_2 > R_1$.

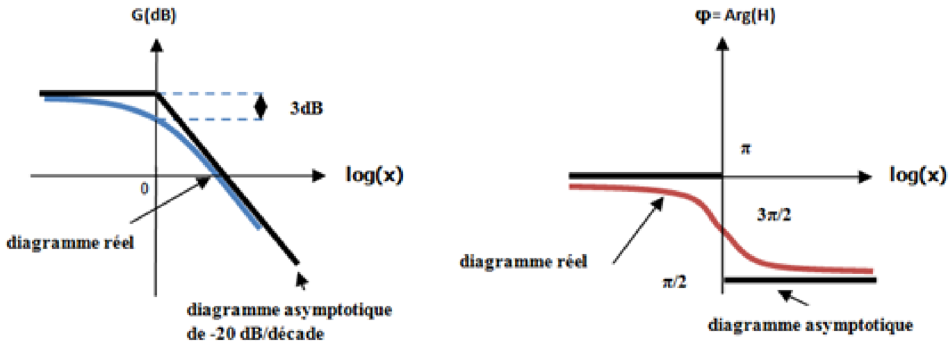


Figure 2.3: Bode diagram first order Low pass filter

First order High Pass Filter

The transfer function of the first order High pass filter is given by,

$$H(jX) = H_0 \frac{jX}{1+jX} \tag{2.2}$$

where, $X = \frac{\omega}{\omega_c}$, ω_c is the cut-off frequency.

Exercise 2.2 Find the transfer function for the circuit shown in Figure 2.4, ■

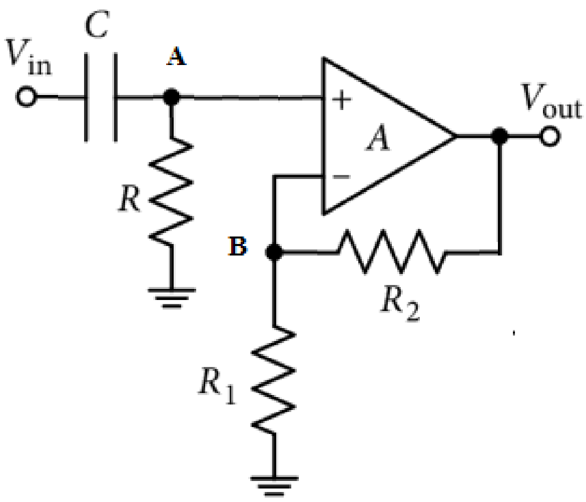


Figure 2.4: First order High pass filter

Solution 2.2

In **A**,

$$V^+ = \frac{R}{R+\frac{1}{jC\omega}} V_{in} = \frac{jRC\omega}{1+jRC\omega} V_{in}$$

In **B**,

$$V^- = \frac{R_1}{R_1+R_2} V_{out}$$

From **A** and **B** we obtain,

$$H(j\omega) = \frac{V_{out}}{V_{in}} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{jRC\omega}{1 + jRC\omega}\right)$$

By identification with equation 2.2 we obtain, $H_0 = 1 + \frac{R_2}{R_1}$ and $\omega_c = \frac{1}{CR}$.

Bode Diagram

Figure 2.5 gives the bode diagram of the filter,

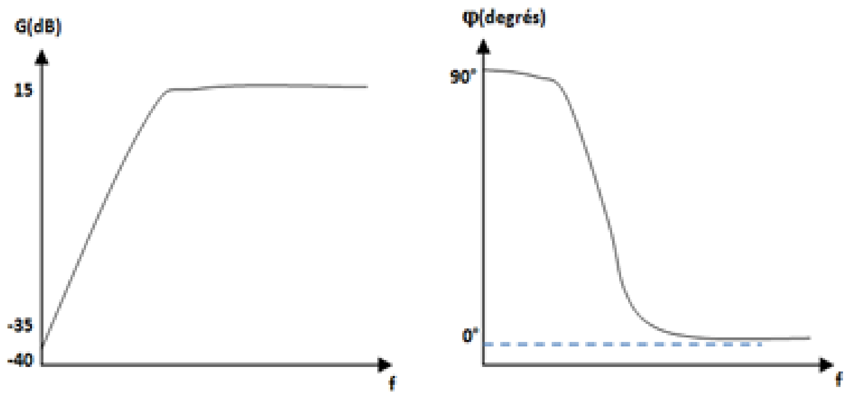


Figure 2.5: Bode diagram of the first order High Pass Filter $R1 = 1k\Omega$, $R2 = 3.3k\Omega$, $R = 4.7k\Omega$, $C = 1\mu F$

2.2.4 Second-order active filters

The transfer function of the second-order filter is given by,

$$H(j\omega) = \frac{\alpha + j\beta x - \gamma x^2}{1 + j\frac{x}{Q} - x^2} \tag{2.3}$$

where, $x = \frac{\omega}{\omega_0}$, and Q is the quality factor, α, β, γ are constants.

Second-order Low Pass filter

The transfer function of the second order Low Pass Filter is given by,

$$H(j\omega) = \frac{H_0}{1 + j\frac{x}{Q} - x^2} \tag{2.4}$$

where, H_0 is static amplification.

Exercise 2.3 Find the transfer function, quality factorand and ω_0 for the circuit shown in Figure 2.6,

Solution 2.3

By using the voltage divider formula we obtain,

$$V_{out} = \frac{\frac{1}{jCw}}{jLw + R + \frac{1}{jCw}} V_{in}$$

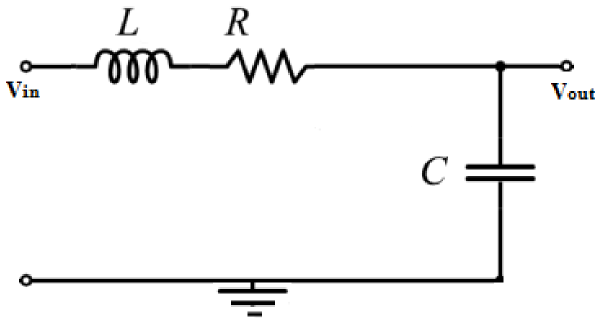


Figure 2.6: Second order Low Pass filter

$$H(jw) = \frac{V_{out}}{V_{in}} = \frac{1}{1 + jRCw - LCw^2}$$

We obtain,

$w_0 = \frac{1}{\sqrt{LC}}$ and $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

- **Gain Bode diagram**

- maximum value of the $|H(j\omega)|$

The magnitude of the transfer function is given by,

$$|H(j\omega)| = \frac{1}{\sqrt{\left(1 - \frac{w^2}{w_0^2}\right)^2 + \left(\frac{w}{Qw_0}\right)^2}}$$

if $Q > \frac{1}{\sqrt{2}}$ **:** the maximum value of the $|H(j\omega)|$ is, $H_{\max} = \frac{2Q^2}{\sqrt{4Q^2-1}}$.

where, $\omega_{\max} = \omega_0\sqrt{1 - \frac{1}{2Q^2}}$, ω is the resonance frequency of the filter.

if $Q < \frac{1}{\sqrt{2}}$ **:** the maximum value of the $|H(j\omega)|$ equal 12 (static amplification, $\omega = 0$).

The study of asymptotes consists in determining the equations of the asymptotic curves in low frequencies and in high frequencies, ie when ω tends to 0 and ∞.

$\omega \to 0$ rad s^{-1} and G $\to$ 0 db

$\omega \to \infty$ and G $\to 40\log(\omega_0) - 40\log(\omega)$ db

we see that the slope of the high frequency asymptote is -40 dB per decad or -12 dB per octave. The BODE diagram is the union of the two asymptotes which intersect in ω_0 as shown in Figure 2.7.

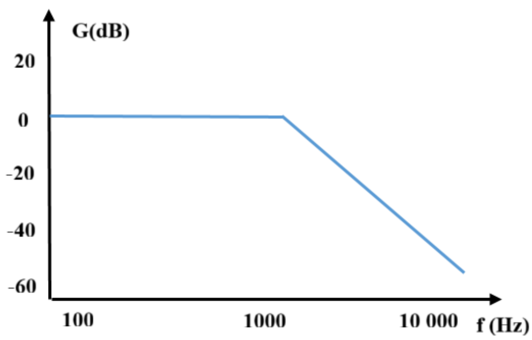


Figure 2.7: BODE diagram Second order Low Pass

The BODE curve for the two values of Q is shown in Figure2.8.

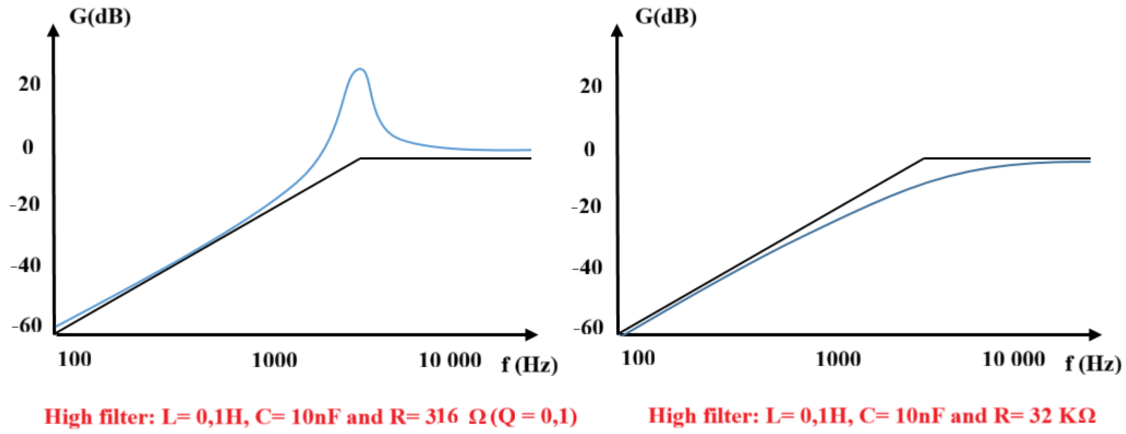


Figure 2.8: BODE diagram Second order Low Pass filter for the two values of Q

- **Phase Bode diagram**
  The study of asymptotes consists in determining the phase values for low and high frequencies ie, when ω tends to 0 and ∞.
  $\omega \to 0$ rad s^{-1}, $\varphi \to -\pi$ rad or $\varphi \to \pi$ rad.
  $\omega \to \infty$, $\varphi \to 0$ rad.
  It is necessary to study the evolution of the phase as a function of frequency. In our case, the argument of a transfer function is $\pm\pi - \arg\left(1 + j\frac{x}{Q} - x^2\right)$. The real part of $\left(1 + j\frac{x}{Q} - x^2\right)$ changes from 1 to $-\infty$ and its imaginary part is always positive. In the complex plane, we are in the top two quadrants. So, the argument of $\left(1 + j\frac{x}{Q} - x^2\right)$ changes from 0 to π, So the phase of the transfer function changes from 0 to π or $-\pi$ to -2π. We have assumed that the phase changes between π and $-\pi$. So,
  if $\omega \to 0$ rad s^{-1}, $\varphi \to \pi$ rad.
  if $\omega \to \infty$, $\varphi \to 0$ rad.
  The transition from one asymptote to another takes place at the ω_0, if, $\omega = \omega_0 \Rightarrow \varphi = \frac{\pi}{2}$ rad as shown in Figure 2.9

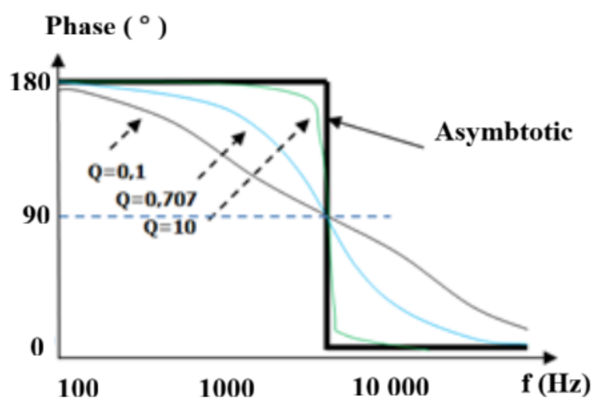


Figure 2.9: Phase diagram Second order Low Pass filter

Second-order Band Pass filter

The transfer function of a second order band pass filter is written in the form,

$$H(j\omega) = \frac{j\frac{x}{Q}H_0}{1+j\frac{x}{Q}-x^2} = \frac{H_0}{1+jQ\left(x-\frac{1}{x}\right)} \quad (2.5)$$

Exercise 2.4 Find the transfer function for the circuit shown in Figure 2.10, ■

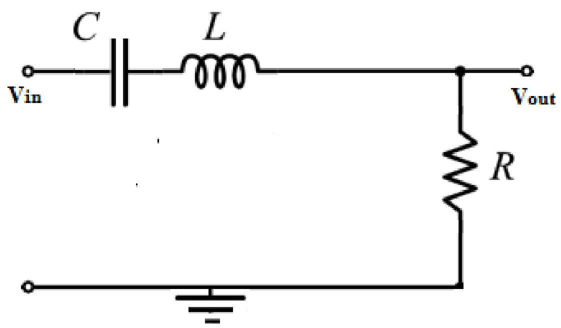


Figure 2.10: Band pass filter

Solution 2.4

$$V_{out} = \frac{R}{jLw+R+\frac{1}{jCw}}V_{in}$$

$$H(jw) = \frac{V_{out}}{V_{in}} = \frac{jRCw}{1+jRCw-LCw^2} = \frac{1}{1+j\left(\frac{L}{R}w-\frac{1}{RCw}\right)}$$

We obtain,
$w_0 = \frac{1}{\sqrt{LC}}$, $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

- **Gain Bode diagram**
  - maximum value of the $|H(j\omega)|$
  The magnitude of the transfer function is given by,

$$|H(j\omega)| = \frac{1}{\sqrt{1+Q^2\left(\frac{w}{w_0}-\frac{w_0}{w}\right)^2}}$$

  The maximum value of $H(j\omega)$ is $H_{max} = 1$ with, $\omega_{max} = \omega_0 \ \forall Q$.
  if $\omega \to 0$ rad s^{-1}, G $\to -20\log(Q\omega_0) + 20\log(\omega)$ dB.
  if $\omega \to \infty$, G $\to 20\log\left(\frac{\omega_0}{Q}\right) - 20\log(\omega)$ dB. The asymptotic Gain plot is the union of the two asymtotes which intersect in $(\omega_0, -20\log(Q))$ as shown in Figure 2.11.
  The Gain curve is shown in Figure 2.12,
- **Phase Bode diagram**
  The study of asymptotes consists in determining the phase values for low and high frequencies ie, when ω tends to 0 and ∞.
  if $\omega \to 0$ rad s^{-1}, $\varphi \to \pi/2$ rad.

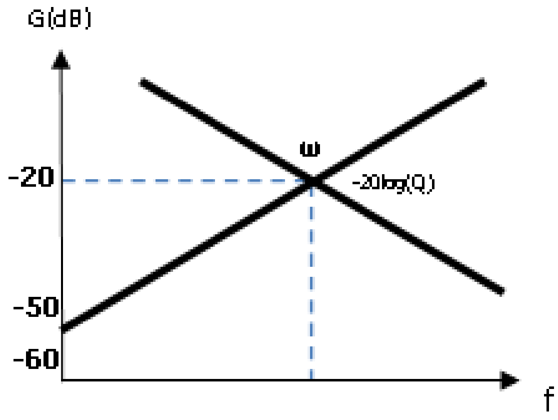


Figure 2.11: Asymptotic Bode diagram of the band-pass filter for $L = 0.1$ H and $C = 10nF$, $Q = 10$

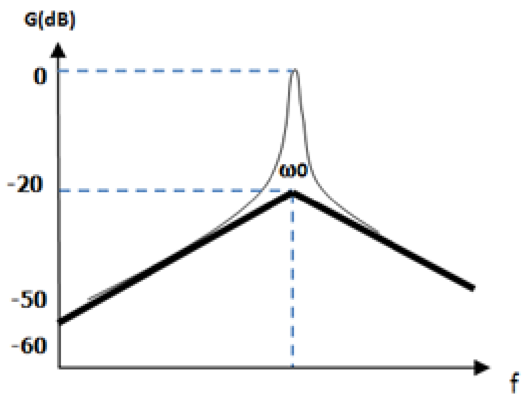


Figure 2.12: Asymptotic Bode diagram of the band-pass filter for $L = 0.1$ H, $C = 10nF$, $R = 316$ ω ($Q = 10$).

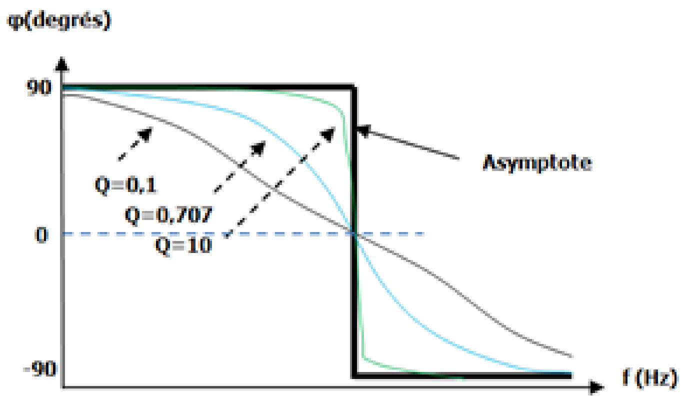


Figure 2.13: phase Bode diagram

if $\omega \to \infty$, $\varphi \to -\pi/2$ rad.
The argument of the transfer function is, $-\arctan(\mathrm{Q}\left(\mathrm{x} - \frac{1}{\mathrm{x}}\right)$. The transition from one asymptote to another takes place at ω_0. The phase Bode diagram is shown in Figure 2.13

- **Bandwidth**
  The bandwidth of a bandpass filter is the pulsation interval given by,
  $BW = [\omega_{c1}, \omega_{c2}]$ or $G(\omega_{ci}) = G(\omega_0) - 3$ dB.
  Second order band pass filter has two cut-off pulsations ω_{c_1} and ω_{c_2},
  $|H(j\omega_{c1})| = |H(j\omega_{c2})| = \frac{1}{\sqrt{2}}$, So, $|H(j\omega_{ci})| = \frac{1}{\sqrt{1+Q^2\left(\frac{\omega_{ci}}{\omega_0}-\frac{\omega_0}{\omega_{ci}}\right)^2}} = \frac{1}{\sqrt{2}}$.

  we can deduce,
  $\mathrm{Q}^2\left(\frac{\omega_{\mathrm{ci}}}{\omega_0}-\frac{\omega_0}{\omega_{\mathrm{ci}}}\right)^2 = 1 \Rightarrow \mathrm{Q}\left(\frac{\omega_{\mathrm{ci}}}{\omega_0}-\frac{\omega_0}{\omega_{\mathrm{ci}}}\right) = \pm 1$. This equation can be written as,

  $$\frac{\omega_{ci}^2}{\omega_0^2} \pm \frac{\omega_{ci}}{Q\omega_0} - 1 = 0$$

  The solutions of this equation are,
  $\omega_{c1} = \frac{\omega_0}{2Q}\left(\sqrt{1+4Q^2}-1\right)$ and $\omega_{c2} = \frac{\omega_0}{2Q}\left(\sqrt{1+4Q^2}+1\right)$.
  The quality factor is,

  $$\frac{1}{Q} = \frac{|\omega_{c2} - \omega_{c1}|}{\omega_0}$$

  This is a coefficient which defines the quality of the band pass filter as shown in Figure 2.14.

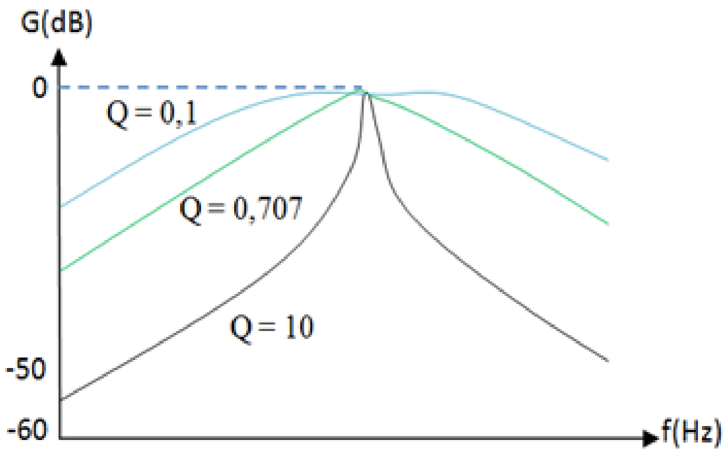


Figure 2.14: Gin plot for different value of Q

Second-order Band Stop filter

The transfer function of a second order band stop filter is given by,

$$H(j\omega) = \frac{(1-x^2)H_0}{1+j\frac{x}{Q}-x^2} \tag{2.6}$$

Example 2.5

Find the transfer function for the circuit shown in Figure 2.15,

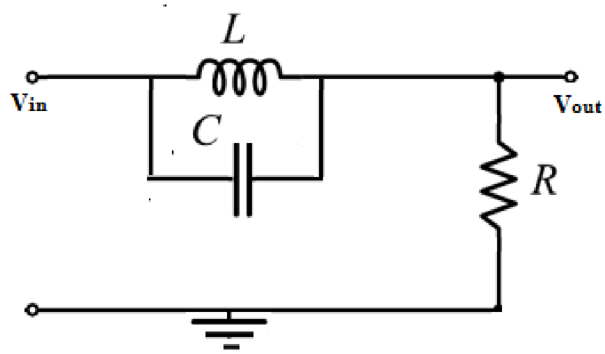


Figure 2.15: Band stop filter

Solution 2.5
By using the voltage divider rule we obtain, $V_{out} = \frac{R}{R+\frac{1}{jCw+\frac{1}{jLw}}} V_{in}$

The transfer function is given by,

$$H(jw) = \frac{V_s}{V_e} = \frac{1-LCw^2}{1+j\frac{L}{R}w-LCw^2}$$

where, $w_0 = \frac{1}{\sqrt{LC}}$ and $Q = R\sqrt{\frac{C}{L}}$.

- **Gain Bode diagram**

  if $\omega \to 0$ rad s^{-1}, G $\to 0$ dB.
  if $\omega \to \infty$, G $\to 0$ dB.
  The Gain Bode diagram is shown in Figure 2.16

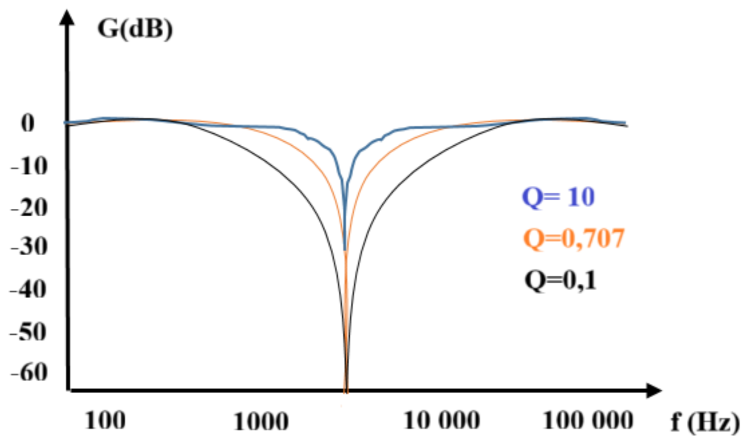


Figure 2.16: Gain Bode Plot Band stop filter

- **Phase Bode diagram**
  The argument of the transfer function is equal to the sum of the argument of $1-x^2$ and the opposite of the complex argument $1+j\frac{x}{Q}-x^2$.
  The argument of $1-x^2$ is is equal to 0 for $\omega < \omega_0$ and π for $\omega > \omega_0$. So the curve has a discontinuity in $\omega = \omega_0$ (See Figure 2.17).

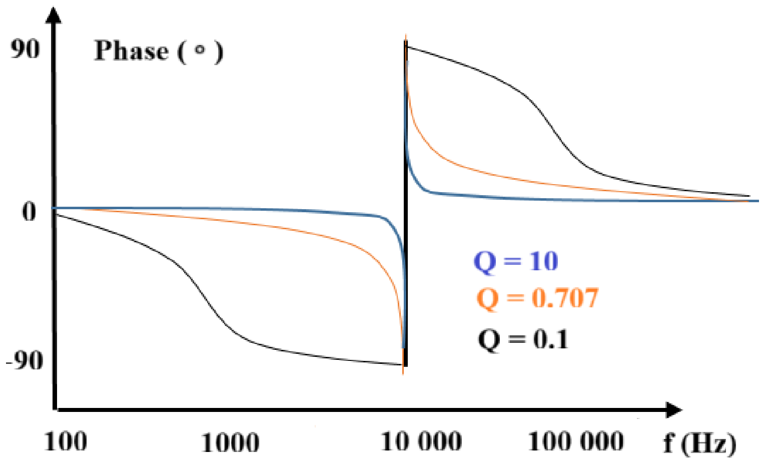


Figure 2.17: Phase Bode Plot Band stop filter

2.3 Oscillator

In this part, a device which works on the principle of *positive feedback* is called an **Oscillator**. An oscillator is a circuit which basically acts as generator, generarating the output signal which oscillates with **constant amplitude** and **desired frequency**. In short, an oscillator is an amplifier, which uses a positive feedback and without any external input signal, This part explains the various types of oscillator circuits.

2.3.1 Basic theory of oscillators

The feedback is a property which allows to feedback the part of the output, to the same circuit as its input. such a feedback is said to be positive whenever the part of the output that is fed back to the amplifier as its input, is in phase with the original input signal applied to the amplifier. Consider a non inverting amplifier with the voltage gain A as shown in Figure 2.18. Assume that a sinusoidal input signal V_s is appled to the circuit. As amplifier is non inverting the output voltage V_o is in phase with the input signal V_s. The part of the output is fed back to the input with the help of feedback

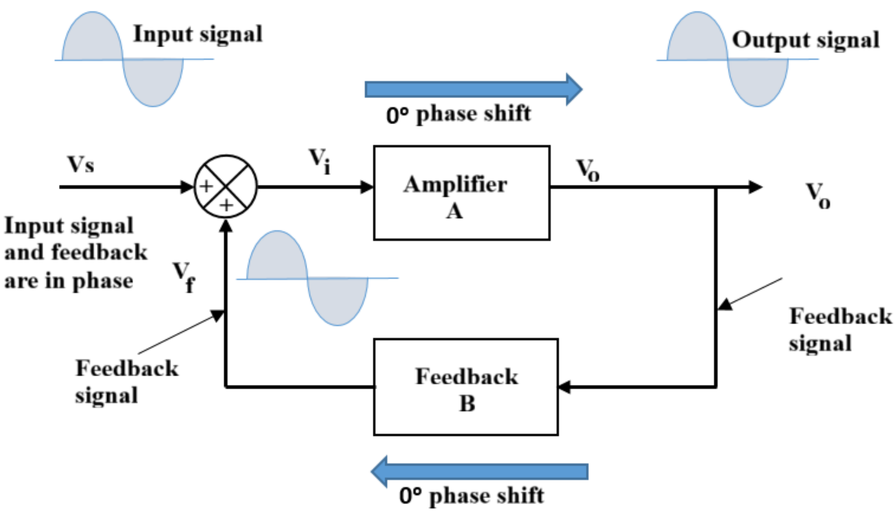


Figure 2.18: Positive feedback

network. No phase change is introduced by the feedback network. Hencee the feedback voltage V_f

is in phase with the input signal V_s. As the phase of the feedback signal is same as that of the input applied, the feedback is **called positive feedback**.

Gain with feedback

The open loop gain A of the amplifier is given as,

$$A = \frac{V_o}{V_i}$$

The ratio of output V_o to input V_s considering effect of feedback is called closed loop gain of the circuit or gain with feedback denoted as A_f .

$$A_f = \frac{V_o}{V_s}$$

The feedback is positive and voltage V_f is added to V_s to generate input of amplifier V_i we can write,

$$V_i = V_s + V_f$$

The feedback voltage V_f depends on the feedback element gain β. So we can write,

$$V_f = \beta V_o$$

So,

$$A_f = \frac{V_o}{V_i - \beta V_o}$$

Dividing both numerator and denominator by V_i

$$A_f = \frac{V_o/V_i}{1 - \beta V_o/V_i}$$

So,

$$A_f = \frac{A}{1 - A\beta}$$

It must be noted that β the feedback network gain is always a fraction and hence $\beta < 1$. So the feedback network is an attenuation network. To start with the oscillations $A\beta > 1$ but the circuit adjusts itself to get $A\beta = 1$, when it produces sinusoidal oscillations while working as an oscillator.

2.3.2 Barkhausen criterion

Consider a basic inverting amplifier with an open loop gain A. The feedback factor β is less than unity. As basic amplifier is inverting. it produces a phase shift of 180° between input and output as shown in Figure 2.19.
The input V_i applied to the amplifier is to be derived from its output V_o using feedback network. But the voltage derived from output using feedback network must be in phase with V_i. thus the feedback network must introduce a phase shift of 180° while feeding back the voltage from output to input. The output voltage is given as,

$$V_o = AV_i$$

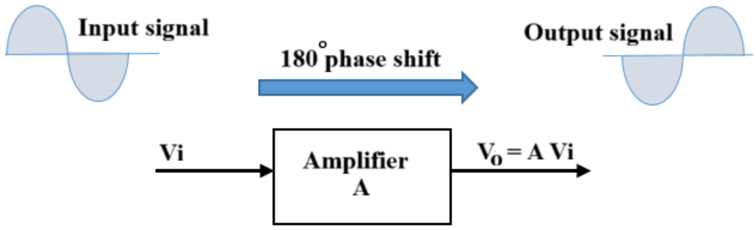


Figure 2.19: Inverting amplifier

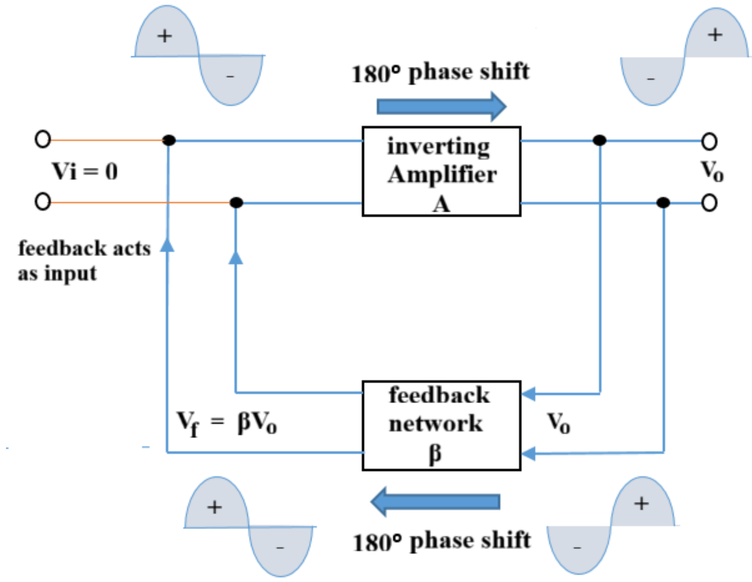


Figure 2.20: Diagram of oscillator

The feedback voltage is,

$$V_f = \beta V_o$$

So,

$$V_f = A\beta V_i$$

For the oscillator, we want that feedback should drive the amplifier and hence V_f must act as V_i, in this case we can write,

$$|A\beta| = 1$$

and the phase of the V_f is same as V_i i.e feedback network should introduce 180° phase shift in addition to 180° introduced by inverting amplifier. this ensures positive feedback. So the total phase shift around a loop is 360°. The two conditions discussed above, required to work the circuit as an oscillator are called **Barkhausen criterion**

The Barkhausen criterion states that,

1. The total phase shift around a loop, as the signal proceeds from input through amplifier, feedback network back to input again, completing a loop, is precisely 0° or 360°.
2. The magnitude of the product of the the open loop gain of the amplifier A and the magnitude of the feedback factor β is unity i.e. $|A\beta| = 1$.

Effect of the magnitude of the $A\beta$

- $|A\beta| > 1$: when the total phase shift around loop is 0° or 360° and $|A\beta| > 1$, then the output oscillates but the oscillations are growing type. The amplitude of oscillations goes on increasing as shown in Figure 2.21

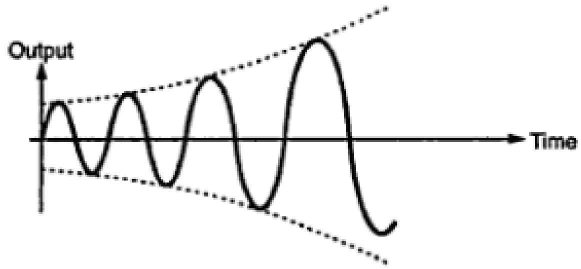


Figure 2.21: Growing type oscillations

- $|A\beta| = 1$: when total phase shift around a loop is 0° or 360° ensuring positive feedback and $|A\beta| = 1$ then the oscillations are with constant frequency and amplitude called sustained oscillations.

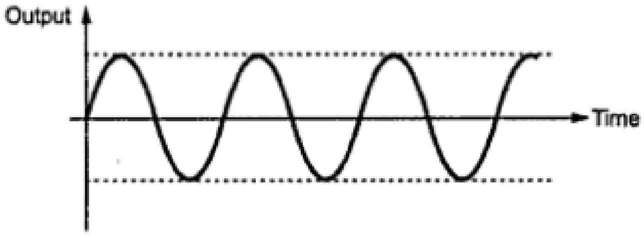


Figure 2.22: Sustained oscillations

- $|A\beta| < 1$: when total phase shift is 0° or 360° but $|A\beta| < 1$ then the oscillations are of decaying type i.e. such oscillation amplitude decreases exponentially and the oscillations finally cease. Thus circuit worck as an amplifier without oscillations as shown in Figure 2.23. So to start the oscillations without input, $|A\beta|$ is kept higher than unity and then circuit adjusts itself to get $|A\beta| = 1$ to result sustained oscillations.

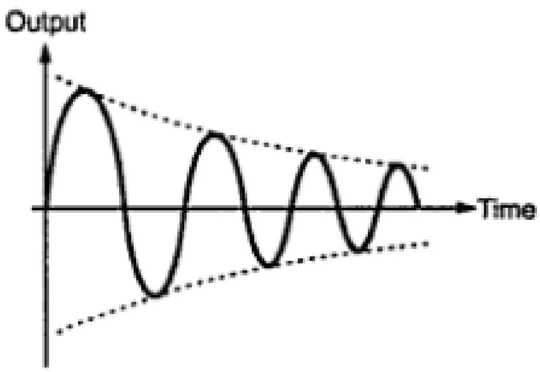


Figure 2.23: Exponentially decaying oscillations

Exercise 2.5 In a certain oscillator circuit, the gain of the amplifier is $\frac{-16.10^6}{j\omega}$ and the feedback factor is $\frac{10^3}{[2.10^3+j\omega]^2}$. Verify the Barkhausen criterion for the sustained oscillations. Also find the frequency at which circuit will oscillate. ■

Solution

From the given information we can write, $A = \frac{-16.10^6}{j\omega}$ and $\beta = \frac{10^3}{[2.10^3+j\omega]^2}$

To verify the Barkhausen criterion means to verify whether $|A\beta| = 1$ at a frequency for which $Ang(A\beta) = 0°$. Let us express, $A\beta$ in its rectangular form.

$$A\beta = -\frac{16*10^6*10^3}{j\omega[2*10^3+j\omega]^2} = -\frac{16*10^9}{j\omega[4*10^6+4*10^3 j\omega+(j\omega)^2]}$$

$$A\beta = -\frac{16*10^9}{4*10^6 j\omega+4*10^3(j\omega)^2-j\omega^3} = -\frac{16*10^9}{j\omega[4*10^6-\omega^2]-[\omega^2*4*10^3]}$$

Rationalising the denominator function we get,

$$A\beta = -\frac{16*10^9[-4*10^3\omega^2-j\omega(4*10^6-\omega^2)]}{[(-4*10^3\omega^2)+j\omega(4*10^6-\omega^2)][(-4*10^3\omega^2)-j\omega(4*10^6-\omega^2)]}$$

Using $(a+b)(a-b) = a^2+b^2$ in the denominator,

$$A\beta = -\frac{16*10^9[-4*10^3\omega^2-j\omega(4*10^6-\omega^2)]}{(-4*10^3\omega^2)^2-(j\omega(4*10^6-\omega^2))^2}$$

$$A\beta = \frac{16*10^9[4*10^3\omega^2+j\omega(4*10^6-\omega^2)]}{16*10^6\omega^4+\omega^2(4*10^6-\omega^2)^2}$$

Now to have **Ang** $A\beta = 0°$, the imaginary part of $A\beta$ must be zero. This is possible when,

$$\omega(4*10^6-\omega^2) = 0$$

$\omega = 0$ or $4*10^6-\omega^2 = 0.$
$\omega^2 = 4*10^6$ neglecting zero value of frequency.
$\omega^2 = 2*10^3$ rad/sec.
At this frequency $|A\beta|$ can be obtained as,

$$|A\beta| = \frac{16*10^9[4*10^3\omega^2]}{16*10^6\omega^4+\omega^2(4*10^6-\omega^2)^2}$$

$$|A\beta| = \frac{16*10^9[4*10^3 4*10^6]}{16*10^6*16*10^{12}+4*10^6(4*10^6-4*10^6)^2}$$

$$|A\beta| = \frac{2.56*10^{20}}{2.56*10^{20}+0} = 1$$

At $\omega = 2*10^3$ rad/sec, **Ang**$A\beta = 0°$ as imaginary part is zero while $|A\beta| = 1$. Thus Barkhausen criterion is satisfied. The frequency at which circuit will oscillate is the value of ω for which $|A\beta| = 1$ and **Ang** $A\beta = 0°$ at the same time,
i.e. $\omega = 2*10^3$ rad/sec.
But $\omega = 2\pi f$.

$$f = \frac{\omega}{2\pi} = \frac{2*10^3}{2\pi} = 318.309Hz$$

2.3.3 RC feedback network

In RC phase shift oscillator, the feedback network consists of three RC sections each producing a phase shift of 60°, thus total phase shift due to feedback is 180°. Such a feedback network is shown in Figure 2.24. The network is also called the ladder network. All the resistance values and

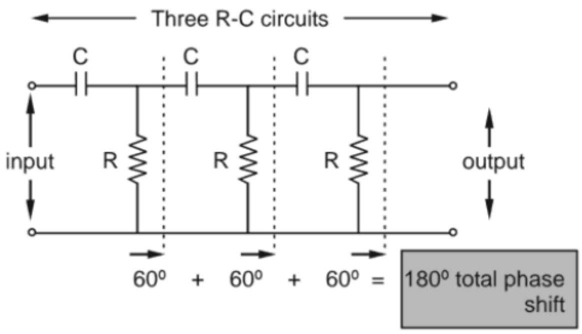


Figure 2.24: RC oscillator

all the capacitance values are the same, so that for a particular frequency, each section of R and C produces a phase shift of 60°.

2.3.4 Phase shift oscillator using Transistor

A common emitter (CE) single stage amplifier is used as a basic amplifier. This produces 180° phase shift. The feedback network consists of 3 RC sections each producing 60° phase shift. Such a RC phase shift oscillator using BJT amplifier is shown in Figure 2.25. The total phase shift around

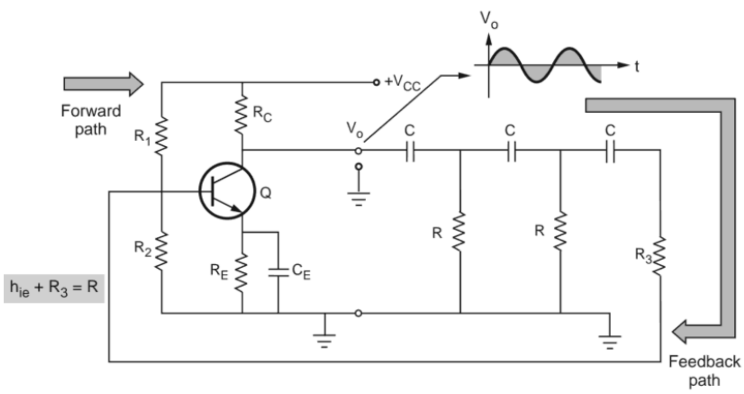


Figure 2.25: BJT phase shift oscillator
[Electron Devices and Circuits De Atul. P. Godse, Uday]

a loop is 180° of amplifier and 180° due to 3 RC sections, thus 360°. This satisfies the required condition for positive feedback and circuit works as an oscillator. The frequency of sustained oscillations generated depends on the values of R and C and given by,

$$f = \frac{1}{2\pi\sqrt{6}RC} Hz$$

Actually to satisfy the Barkhausen condition, the expression for the frequency of oscillations is given by,

$$f = \frac{1}{2\pi RC}\frac{1}{\sqrt{6+4K}} Hz$$

where, $K = R_c/R$, as practically R_c/R is small, K is neglected. The condition of h_{fe} for the transistor to obtain the oscillations is given by,

$$h_{fe} > 4K + 23 + \frac{29}{K}$$

and value of K for minimum h_{fe} is 2.7 hence minimum $h_{fe} = 44.5$. So, transistor with h_{fe} less than 44.5 cannot be used in phase shift oscillator. But for most of practical circuits, the expression for the frequency is considered as,

$$f = \frac{1}{2\pi\sqrt{6}RC}$$

Frequency of oscillations

Remplacing the transistor by its approximate h-parameter model, we get the equivalent oscillator circuit as shown in the Figure 2.26. Practically R_3 is used such that h_{ie} of transistor alongwith R_3

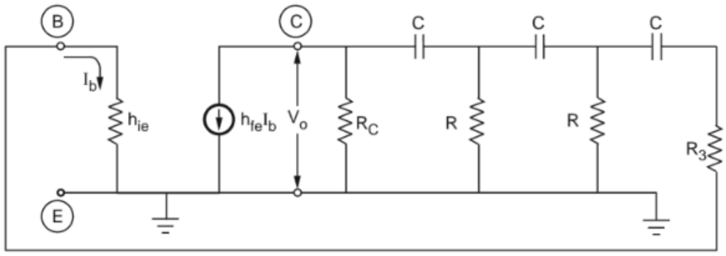


Figure 2.26: Equivalent circuit using h-parameter model

completes the need of R,

$$R = h_{ie} + R_3$$

If the resistances R_1 and R_2 are not neglected then the input impedance of the amplifier stage becomes as,

$$R_i^{'} = R_1 || R_2 || h_{ie}$$

In such a case, the value of R_3 must be so selected that,

$$R_i^{'} + R_3 = R$$

We can replace the current source $h_{fe}I_b$ by its voltage source and assume the ratio of the resistance to R_c to R be K.

$$K = \frac{R_c}{R}$$

The modified equivalent circuit is shown in the Figure 2.27. Applying KVL for the various loops in the modified equvalent circuit we get,

For Loop 1

$$-I_1 R_c - \frac{1}{j\omega C} I_1 - I_1 R + I_2 R - h_{fe} I_b R_c = 0$$

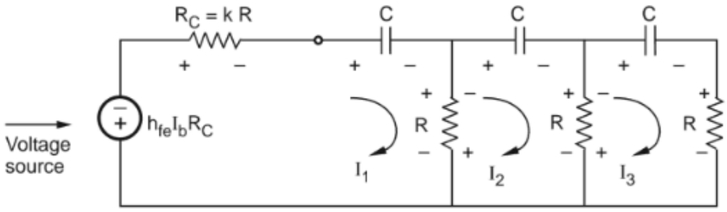


Figure 2.27: Modified equivalent circuit model

Replacing R_c by KR and $j\omega$ by s we get,

$$I_1[(K+1)R+\frac{1}{sC}]-I_2R=-h_{fe}I_bKR \tag{2.7}$$

For Loop 2

$$\begin{aligned}&-\frac{1}{j\omega C}I_2-I_2R-I_2R+I_1R+I_3R=0\\&-I_1R+I_2[2R+\frac{1}{sC}]-I_3R=0\end{aligned} \tag{2.8}$$

For Loop 3

$$\begin{aligned}&-I_3\frac{1}{j\omega C}-I_3R-I_3R+I_2R=0\\&-I_2R+I_3[2R+\frac{1}{sC}]=0\end{aligned} \tag{2.9}$$

Using Cramer's rule to solve for I_3,

$$D=\begin{vmatrix}(K+1)R+\frac{1}{sC} & -R & 0\\ -R & 2R+\frac{1}{sC} & -R\\ 0 & -R & 2R+\frac{1}{sC}\end{vmatrix}$$

$$D=[(K+1)R+\frac{1}{sC}][2R+\frac{1}{sC}]^2-R^2[2R+\frac{1}{sC}]-R^2[(K+1)R+\frac{1}{sC}]$$
$$D=\frac{[sRC(K+1)+1][2sCR+1]^2}{sC^3}-\frac{R^2(2sCR+1)}{sC}-\frac{R^2[(K+1)sRC+1]}{sC}$$

First term can be written as,

$$=\frac{[sKRC+sRC+1][4s^2C^2R^2+4sRC+1]}{s^3C^3}$$
$$=\frac{4s^4KR^3C^3+4s^3R^3C^3+4s^2R^2C^2+4s^2KR^2C^2+4s^2R^2C^2+4sRC+sKRC+sRC+1}{s^3C^3}$$

Second and the Third term can be combined to get,

$$=\frac{-R^2[KsRC+sRC+1]-R^2[1+2sRC]}{sC}$$
$$=\frac{-[2R^2+3sR^3C+KsR^3C]}{sC}$$

Combining the tow terms and taking LCM as s^3C^3 we get,

$$\begin{aligned} D &= \frac{s^3C^3R^3[4K+4]+s^2C^2R^2[4K+8]+sRC[5+K]+1-[2R^2+3sR^3C+KsR^3C]s^2C^2}{s^3C^3} \\ &= \frac{s^3C^3R^3[3K+1]+s^2C^2R^2[4K+6]+sRC[5+K]+1}{s^3C^3} \end{aligned} \tag{2.10}$$

Now,

$$\begin{aligned} D3 &= \begin{vmatrix} (K+1)R+\frac{1}{sC} & -R & -h_{fe}I_bKR \\ -R & 2R+\frac{1}{sC} & 0 \\ 0 & -R & 0 \end{vmatrix} \\ D_3 &= -R^2(h_{fe}I_bKR) = -KR^3h_{fe}I_b \end{aligned} \tag{2.11}$$

$$I_3 = \frac{D_3}{D} = \frac{-KR^3h_{fe}I_bs^3C^3}{s^3C^3R^3[3K+1]+s^2C^2R^2[4K+6]+sRC[5K+1]+1} \tag{2.12}$$

Now,
I_3, Output current of the feedback circuit.
I_b, Input current of the amplifier.
$I_c = h_{fe}I_b$, Input current of the feedback circuit.
$\beta = \frac{I_3}{h_{fe}I_b}$.
$A = \frac{I_3}{I_b} = h_{fe}$.

$$A\beta = \frac{I_3}{h_{fe}I_b} * h_{fe} = \frac{I_3}{I_b} \tag{2.13}$$

Using equation 2.12 we get,

$$A\beta = \frac{-KR^3h_{fe}s^3C^3}{s^3C^3R^3[3K+1]+s^2C^2R^2[4K+6]+sRC[5K+1]+1} \tag{2.14}$$

Substituting $s = j\omega$ in the equation 2.14 we get,

$$A\beta = \frac{-j\omega^3KR^3h_{fe}C^3}{-j\omega^3C^3R^3[3K+1]-\omega^2C^2R^2[4K+6]+j\omega RC[5+K]+1}$$

Separating the real and imaginary parts in the denominator we get,

$$A\beta = \frac{-j\omega^3KR^3h_{fe}C^3}{[1-4K\omega^2C^2R^2-6\omega^2C^2R^2]-j\omega[3K\omega^2R^3C^3+\omega^2R^3C^3-5RC-KRC]}$$

Dividing numerrator and denominator by $j\omega^3R^3C^3$,

$$A\beta = \frac{Kh_{fe}}{[\frac{1-4K\omega^2C^2R^2-6\omega^2C^2R^2}{-j\omega^3R^3C^3}]-[\frac{j\omega(3K\omega^2R^3C^3+\omega^2R^3C^3-5RC-KRC)}{-j\omega^3R^3C^3}]}$$

$$A\beta = \frac{Kh_{fe}}{j[\frac{1}{\omega^3R^3C^3}-\frac{4K}{\omega RC}-\frac{6}{\omega RC}]+[3K+1-\frac{5}{\omega^2R^2C^2}-\frac{K}{\omega^2R^2C^2}]}$$

Replacing $\alpha = \frac{1}{\omega RC}$ we obtain,

$$A\beta = \frac{Kh_{fe}}{[3K+1-5\alpha^2-K\alpha^2]+j[\alpha^3-4K\alpha-6\alpha]} \tag{2.15}$$

As per the Barkhausen condition, **Ang** $A\beta = 0°$. the angle of numerator term Kh_{fe} of equation 2.15 is 0° hence to have angle of $A\beta$ term as 0°, the imaginary part of the denominator term must be 0.

$$\begin{aligned}
\alpha^3-4K\alpha-6\alpha &= 0\\
\alpha(\alpha^2-4K-6) &= 0\\
\alpha^2 &= 4K+6\\
\alpha &= \sqrt{4K+6}\\
\frac{1}{RC\omega} &= \sqrt{4K+6}\\
f &= \frac{1}{2\pi RC\sqrt{4K+6}}
\end{aligned} \tag{2.16}$$

This is the frequency at which **Ang** $A\beta = 0°$. At the same frequency, $|A\beta| = 1$. Substituting $\alpha = \sqrt{4K+6}$ in the equation 2.15 we get,

$$A\beta = \frac{Kh_{fe}}{3K+1-(4K+6)(5+K)} = \frac{Kh_{fe}}{3K+1-20K-30-4K^2-6K} = \frac{Kh_{fe}}{-4K^2-23K-29}$$

Now, $|A\beta| = 1 \Rightarrow Kh_{fe} = 4K^2+23K+29$.

$$h_{fe} = 4K+23+\frac{29}{K} \tag{2.17}$$

This must be the value of h_{fe} for the oscillations.

Minimum value of h_{fe}

To get minimum value of h_{fe},

$$\frac{dh_{fe}}{dK} = \frac{d}{dK}[4K+23+\frac{29}{K}] = 0$$

$$[4-\frac{29}{K^2]} = 0$$

$$K = \frac{29}{4} = 2.6925$$

$$(h_{fe})_{min} = 4(2.6925)+23+\frac{29}{2.6925} = 44.54 \tag{2.18}$$

Key point : Thus for the circuit to oscillate, we must select the transistor whose h_{fe} min should be greater than 44.54.

By changing the values of R and C, the frequency of the oscillator can be changed. But the values of R and C of all three sections must be changed simultaneously to satisfy the oscillating conditios. But this is practically impossible. Hence the phase shift oscillator is considered as a fixed frequency oscillator.

2.3.5 Wien bridge oscillator

Generaly in an oscillator, amplifier stage introduces 180° phase shift, to obtain a phase shift of 360° around a loop. This is required condition for any oscillator. But Wien oscillator uses a non-inverting amplifier and hence does not provides any phase shift during amplifier stage. As total phase shift required is 0° or $2n\pi$ rad, in Wien type no phase shift is necessary through feedback. Abasic Wien in this oscillator and an amplifier stage is shown in Figure 2.28. The output of the

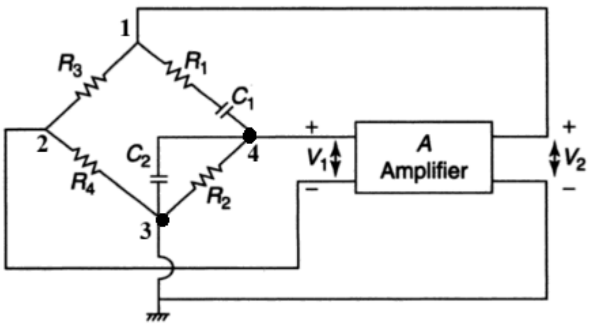


Figure 2.28: Wien oscillator

amplifier is applied between the terminals 1 and 3, which is the input to feedback network. While the amplifier input is supplied from the diagonal terminals 2 and 4, which is the output from the feedback network. Thus amplifier supplied its own input through the Wien bridge as a feedback network. The two arms of the bridge, namely R_1, C_1 in series and R_2, C_2 in parallel are called *frequency sensitive arms*. This is because the components of thes two arms decide the frequency of the oscillator. Let us find out the gain of the feedback network. As seen earlier input V_{in} to the feedback network is between is 1 and 3 while V_f of the feedback networck is between 2 and 4. This is shown in Figure 2.29. Such a feedback network is *called lead-lag network*. This is because at

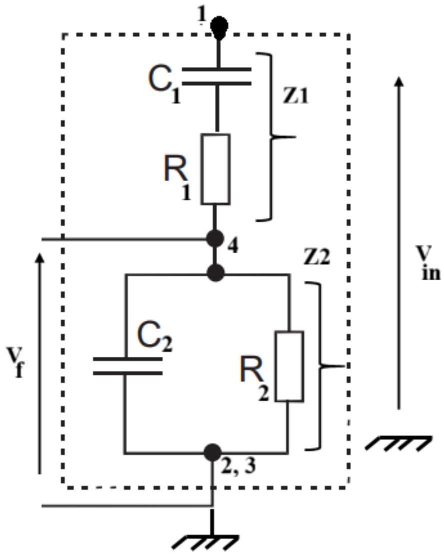


Figure 2.29: Feedback network of Wien oscillator

very low frequencies it acts like a lead while at very high frequencies it acts like lag network.

Frequency of oscillations

Now from the Figure 2.29 as shown,

$$Z_1 = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}$$

$$Z_2 = R_2 || \frac{1}{j\omega C_2} = \frac{R_2 * \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}} = \frac{R_2}{1 + j\omega R_2 C_2} \quad (2.19)$$

Replacing $s = j\omega$,

$$Z_1 = \frac{1 + sR_1 C_1}{sC_1}$$

and,

$$Z_2 = \frac{R_2}{1 + sR_2 C_2}$$

$$I = \frac{V_{in}}{Z_1 + Z_2}$$

and,

$$V_f = IZ_2$$

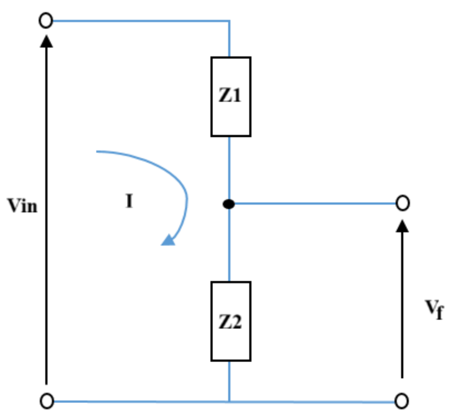


Figure 2.30: Simplified circuit

$$V_f = \frac{V_{in} Z_2}{Z_1 + Z_2}$$

$$\beta = \frac{V_f}{V_{in}} = \frac{Z_2}{Z_1 + Z_2} \quad (2.20)$$

Substituting the values of Z_1 and Z_2,

$$\beta = \frac{\frac{R_2}{1+sR_2C_2}}{[\frac{1+sR_1C_1}{sC_1}]+[\frac{R_2}{1+sR_2C_2}]}$$
$$\beta = \frac{sC_1R_2}{(1+sR_1C_1)(1+sR_2C_2)+sC_1R_2}$$
$$\beta = \frac{sC_1R_2}{1+s(R_1C_1+R_2C_2)+s^2R_1R_2C_1C_2+sC_1R_2}$$
$$\beta = \frac{sC_1R_2}{1+s(R_1C_1+R_2C_2+C_1R_2)+s^2R_1R_2C_1C_2}$$

Replacing s by $j\omega$,

$$\beta = \frac{j\omega C_1R_2}{(1-\omega^2R_1R_2C_1C_2)+j\omega(R_1C_1+R_2C_2+R_2C_1)} \tag{2.21}$$

Rationalising the expression,

$$\begin{aligned}\beta &= \frac{j\omega C_1R_2[(1-\omega^2R_1R_2C_1C_2)-j\omega(R_1C_1+R_2C_2+R_2C_1)]}{(1-\omega^2R_1R_2C_1C_2)^2+\omega^2(R_1C_1+R_2C_2+R_2C_1)^2}\\ \beta &= \frac{\omega^2C_1R_2(R_1C_1+R_2C_2+R_2C_1)+j\omega C_1R_2(1-\omega^2R_1R_2C_1C_2)}{(1-\omega^2R_1R_2C_1C_2)^2+\omega^2(R_1C_1+R_2C_2+R_2C_1)^2}\end{aligned} \tag{2.22}$$

To have zero phase shift of the feedback network, its imaginary part must be zero,

$$\begin{aligned}\omega(1-\omega^2R_1R_2C_1C_2) &= 0\\ \omega^2 &= \frac{1}{R_1R_2C_1C_2}\\ \omega &= \frac{1}{\sqrt{R_1R_2C_1C_2}}\\ f &= \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}\end{aligned} \tag{2.23}$$

This is the frequency of the oscillator and it shows that the components of the frequency sensitive arms are the deciding factors, for the frequency. In practice, $R_1 = R_2 = R$ and $C_1 = C_2 = C$ are selected.

$$f = \frac{1}{2\pi\sqrt{R^2C^2}} = \frac{1}{2\pi RC} \tag{2.24}$$

At $R_1 = R_2 = R$ and $C_1 = C_2 = C$, the gain of the feedback network becomes,

$$\beta = \frac{\omega^2RC(3RC)+j\omega RC(1-\omega^2R^2C^2)}{(1-\omega^2R^2C^2)+\omega^2(3RC)^2}$$

Substituting, $f = \frac{1}{2\pi RC}$ i.e. $\omega = \frac{1}{RC}$, we get the magnitude of the feedback network at the resonating frequency of the oscillator as,

$$\begin{aligned}\beta &= \frac{3}{0+\frac{1}{R^2C^2}*(3RC)^2} = \frac{3}{9}\\ \beta &= \frac{1}{3}\end{aligned} \tag{2.25}$$

The positive sign of β indicate that the phase shift by the feedback is 0°. Now to satisfy the Barkhausen condition for the sustained oscillations, we can write,

$$|A\beta| \geq 1 \Rightarrow |A| \geq \frac{1}{|\beta|} \geq \frac{1}{\frac{1}{3}}$$

$$|A| \geq 3$$

This is the required gain of the amplifier stage , without any phase shift. If $R_1 \neq R_2$ and $C_1 \neq C_2$ then,

$$f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

Substituting in the equation 2.22 we get,

$$\beta = \frac{C_1 R_2}{R_1 C_1 + R_2 C_2 + R_2 C_1}$$

$$|A\beta| \geq 1$$

$$A \geq \frac{R_1 C_1 + R_2 C_2 + R_2 C_1}{C_1 R_2}$$

An important advantage of the Wien bridge oscillator is that by varying the two capacitor values simultaneously by mounting them on the common shaft, different frequency ranges can be provides.

2.3.6 Wien Oscillator using Op-amp

Figure 2.31 shows the Wien bridge circuit using op-amp. The resistance R and capacitor C are the

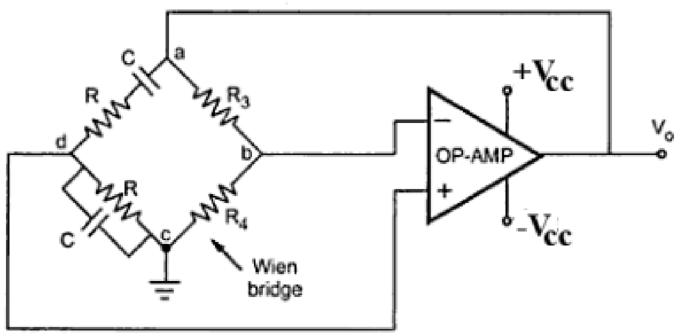


Figure 2.31: Wien oscillator using op-amp

components of frequency sensitive arms of the bridge. The resistance R_3 and R_4 from the part of the feedback path. The op-amp output is connected to bridge input points **a** and **c** while bridge output points **b** and **d** atre connected to op-amp input. The gain of the op can be adjusted by using the resistances R_3 and R_4. The gain of the op is given by,

$$A = 1 + \frac{R_3}{R_4} \tag{2.26}$$

According to the oscillating conditions, $A \geq 3$.

$$\frac{R_3}{R_4} \geq 2 \tag{2.27}$$

Thus the ratio of R_3 and R_4 greater than or equal to two, will provide suffcient loop gain for circuit to oscillate at the frequency calculated as,

$$f = \frac{1}{2\pi RC}$$

Key point: The op-amp is used in the non inverting amplifier to ensure the zero phase shift.

Exercise 2.6 Determine whether the circuit shown in Fig 2.32, will work as an oscillator or not. If yes, determine the frequency of the oscillator. ■

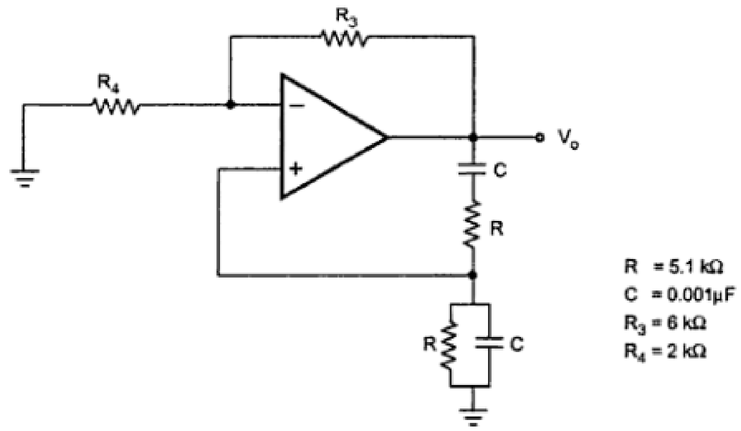


Figure 2.32: Wien oscillator

Solution

The circuit is Wien bridge oscillator using op-amp. The gain of the op-amp is,

$$A = 1 + \frac{R_3}{R_4} = 1 + \frac{6}{2} = 4$$

So, $A > 3$. This satisfies the required oscillating condition. The feedback is given to non-inverting terminal ensuring the zero phase shift. Hence the circuit will work as the oscillator.

$$f = \frac{1}{2\pi RC} = \frac{1}{2\pi * 5.1 * 10^3 * 0.001 * 10^{-6}} = 31.2068 KHz$$

This will be the frequency of oscillations.

2.3.7 Hartley oscillator using BJT transistor

The practical circuit is shown in the Figure 2.33. The resistances R_1 and R_2 are the biasing resistances. The RFC is the radio frequency choke. Its reactance value is very high for high frequencies, hence it can be treated as open circuit. While for d.c conditions, the reactance is zero hence causes no problem for d.c. capacitors. Hence due to RFC, the isolation between a.c. and d.c. operation is achieved. R_E is also a biasing ciruit resistance and C_E is the emetter bypass capacitor. C_{C_1} and C_{C_2} are the coupling capacitor. The common emitter amplifier provides a phase shift of 180°. As emitter is grounded the base and the collector voltages are out of phase by 180°. As the centre of L_1 and L_1 is grounded, when upper end becomes positive, the lower becomes negative and viceversa. So the LC feedback network gives an additional phase shift of 180°, necessary to satisfy oscillation conditions.

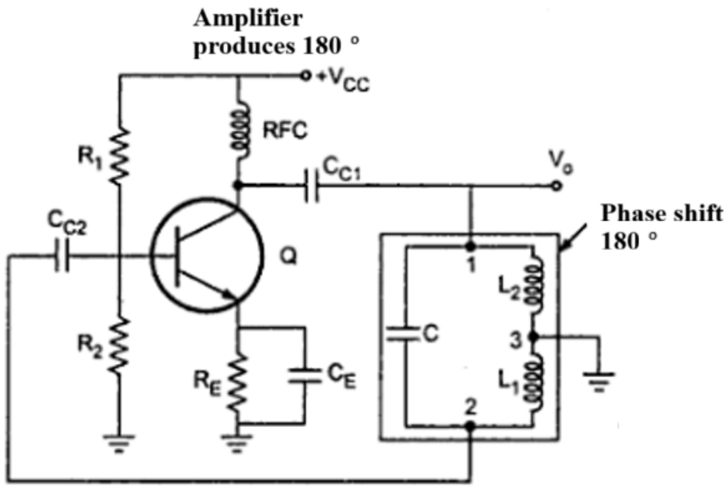


Figure 2.33: Hartley oscillator

Frequency of oscillations

The output current which is the collector current is $h_{fe}I_b$ where I_b is the base current. Assuming coupling condensers are short, the capacitor C is between base ans collector. The inductance L_1 is between base and emitter while the inductance L_2 is between collector and emitter. The equivalent circuit of the feedback is shown in the Figure 2.34. As h_{ie} is the input impedance of the transistor.

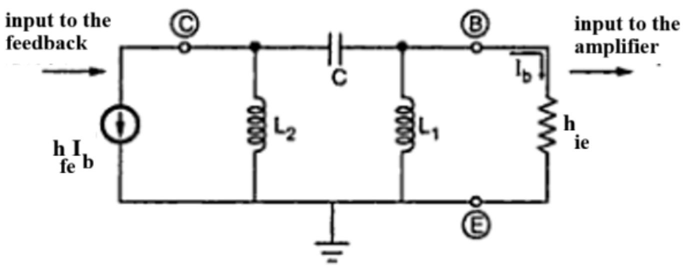


Figure 2.34: Equivalent circuit

The output of the feedback is the current I_b which is the input current of the transistor. While input to the feedback is the output of the transistor which is $I_c = h_{fe}I_b$, converting current source into voltage source we get,

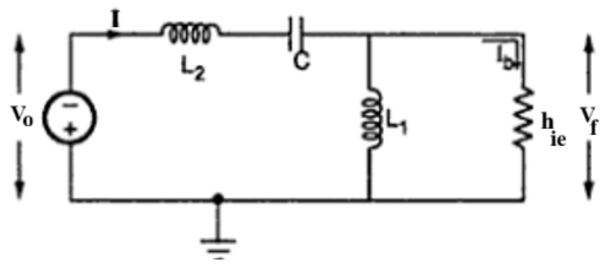


Figure 2.35: Simplified equivalent circuit

$$V_o = h_{fe}I_bX_{L2} = h_{fe}I_bj\omega L_2 \tag{2.28}$$

Now L_1 and h_{ie} are in parallel, so the total current I drawn from the supply is,

$$I = \frac{-V_o}{[X_{L2} + X_c][X_{L1} \| h_{ie}]} \tag{2.29}$$

Key point : Nigative sign, as current direction shown in opposite to the polarities of V_o.

$$X_{L2} + X_c = j\omega L_2 + \frac{1}{j\omega C}$$

and,

$$X_{L1} \| h_{ie} = \frac{j\omega L_1 h_{ie}}{j\omega L_1 + h_{ie}}$$

Substituting in the equation 2.30 we get,

$$I = \frac{-h_{fe} I_b j\omega L_2}{[j\omega L_2 + \frac{1}{j\omega C}] + [\frac{j\omega L_1 h_{ie}}{j\omega L_1 + h_{ie}}]} \tag{2.30}$$

Replacing $j\omega$ by s,

$$I = \frac{-h_{fe} I_b s L_2}{[sL_2 + \frac{1}{sC}] + [\frac{sL_1 h_{ie}}{sL_1 + h_{ie}}]}$$
$$I = \frac{-h_{fe} I_b s L_2}{[\frac{1 + s^2 L_2 C}{sC}] + [\frac{sL_1 h_{ie}}{sL_1 + h_{ie}}]}$$
$$I = \frac{-s h_{fe} I_b L_2 (sC)(sL_1 + h_{ie})}{[1 + s^2 L_2 C][sL_1 + h_{ie}] + sC(sL_1 h_{ie})}$$
$$I = \frac{-s^2 h_{fe} I_b L_2 C (sL_1 + h_{ie})}{s^3 L_1 L_2 C + sL_1 + h_{ie} + s^2 L_2 C h_{ie} + s^2 L_1 C h_{ie}}$$
$$I = \frac{-s^2 h_{fe} I_b L_2 C (sL_1 + h_{ie})}{s^3 L_1 L_2 C + s^2 C h_{ie}(L_1 + L_2) + sL_1 + h_{ie}}$$

According to current division in parallel circuit,

$$I_b = I * \frac{X_{L1}}{X_{L1} + h_{ie}}$$
$$I_b = I * \frac{j\omega L_1}{j\omega L_1 + h_{ie}} \tag{2.31}$$
$$I_b = I * [\frac{sL_1}{sL_1 + h_{ie}}]$$

Substituting value of I from equation 2.30 in equation 2.31,

$$I_b = \frac{-s^2 h_{fe} I_b L_2 C (sL_1 + h_{ie})}{s^3 L_1 L_2 C + s^2 C h_{ie}(L_1 + L_2) + sL_1 + h_{ie}} * [\frac{sL_1}{sL_1 + h_{ie}}]$$
$$I_b = \frac{-s^3 h_{fe} I_b L_1 L_2 C}{s^3 L_1 L_2 C + s^2 C h_{ie}(L_1 + L_2) + sL_1 + h_{ie}} \tag{2.32}$$
$$1 = \frac{-s^3 h_{fe} L_1 L_2 C}{s^3 L_1 L_2 C + s^2 C h_{ie}(L_1 + L_2) + sL_1 + h_{ie}}$$

Substituting $s = j\omega$,

$$1 = \frac{j\omega^3 h_{fe} L_1 L_2 C}{[h_{ie} - \omega^2 C h_{ie}(L_1 + L_2)] + j\omega L_1 (1 - \omega^2 L_2 C)} \tag{2.33}$$

Rationalising the R.H.S of above equation,

$$\begin{aligned} 1 &= \frac{j\omega^3 h_{fe} C L_1 L_2 [h_{ie} - \omega^2 C h_{ie}(L_1 + L_2) - j\omega L_1 (1 - \omega^2 L_2 C)]}{[h_{ie} - \omega^2 C h_{ie}(L_1 + L_2)]^2 + \omega^2 {L_1}^2 (1 - \omega^2 L_2 C)^2} \\ 1 &= \frac{\omega^4 h_{fe} {L_1}^2 L_2 C (1 - \omega^2 L_2 C) + j\omega^3 h_{fe} L_1 L_2 C [h_{ie} - \omega^2 C h_{ie}(L_1 + L_2)]}{[h_{ie} - \omega^2 C h_{ie}(L_1 + L_2)]^2 + \omega^2 {L_1}^2 (1 - \omega^2 L_2 C)^2} \end{aligned} \tag{2.34}$$

To satisfy this equation, imaginary part of R.H.S must bezero.

$$\begin{aligned} \omega^3 h_{fe} L_1 L_2 C [h_{ie} - \omega^2 C h_{ie}(L_1 + L_2)] &= 0 \\ \omega^3 h_{ie} h_{fe} L_1 L_2 C [1 - \omega^2 C (L_1 + L_2)] &= 0 \\ 1 - \omega^2 C (L_1 + L_2) &= 0 \\ \omega^2 &= \frac{1}{C(L_1 + L_2)} \\ \omega &= \frac{1}{\sqrt{C(L_1 + L_2)}} \\ f &= \frac{1}{2\pi\sqrt{C(L_1 + L_2)}} \end{aligned} \tag{2.35}$$

This is the frequency of the oscillations. At this frequency, the restriction of the value of h_{fe} can be obtained, by equating the magnitudes of the both sides of the equation 2.34.

$$\begin{aligned} 1 &= \frac{\omega^4 h_{fe} {L_1}^2 L_2 C (1 - \omega^2 L_2 C)}{0 + \omega^2 {L_1}^2 (1 - \omega^2 L_2 C)^2} \\ 1 &= \frac{h_{fe} L_2}{1 - \omega^2 L_2 C} \\ 1 &= \frac{h_{fe} L_2}{1 - \dfrac{L_2 C}{C(L_1 + L_2)}} = \frac{h_{fe} L_2}{L_1} \\ h_{fe} &= \frac{L_1}{L_2} \end{aligned} \tag{2.36}$$

This is the value of h_{fe}, required to satisfy the oscillating conditions. For a mutual inductance of M,

$$h_{fe} = \frac{L_1 + M}{L_2 + M} \tag{2.37}$$

Now $L_1 + L_2$ is the equivalent inductance of the two inductances L_1 and L_2, connected in series denoted as,

$$L_{eq} = L_1 + L_2 \tag{2.38}$$

Hence the frequency of oscillations is given by,

$$f = \frac{1}{2\pi\sqrt{C L_{eq}}} \tag{2.39}$$

So if the capacitor C is kept variable, frequency can be varied over a large range as per the requirement. In practice, L_1 and L_2 may be wound on a single core that there exists a mutual inductance is considred while determining the equivalent inductance L_{eq}, Hence,

$$L_{eq} = L_1 + L_2 + 2M \tag{2.40}$$

If L_1 and L_1 are assisting each other then sign of 2M is positive while if L_1 and L_2 are in series opposition then sign of 2M is negative.

Example 2.8

In a transistorised Hartley oscillator the two inductances are $2mH$ and $20\mu H$ while the frequency is to be changed from $950KHz$ to $2050KHz$. Calculate the range over which the capacitor is to be varied.

Solution 2.8

The frequency is given by,

$$f = \frac{1}{2\pi\sqrt{CL_{eq}}}$$

where,

$$L_{eq} = L_1 + L_2 = 2 * 10^{-3} + 20 * 10^{-6} = 0.00202KHz$$

for,

$$f = f_{max} = 2050KHz$$

$$2050 * 10^3 = \frac{1}{2\pi\sqrt{C * 0.00202}}$$

$$C = 2.98pF$$

For,

$$f = f_{min} = 950kHz$$

$$950 * 10^3 = \frac{1}{2\pi\sqrt{C * 0.00202}}$$

$$C = 13.89pF$$

Hence C must be varied from $2.98pF$ to $13.89pF$, to get the required frequency variation.

2.3.8 Colpitts oscillator using BJT

The basic circuit is same as transistorised Hartley oscillator, except the tank circuit. The common emitter amplifier causes a phase shift of 180°, while the tank circuit adds further 180° phase shift, to satisfy the oscillating conditions.

Frequency of oscillations

As seen earlier, the output current I_c which is $h_{fe}I_c$ acts as input to the feedback network. While the base current I_b acts as the output current of the tank circuit, flowing through the input impedance of the amplifier h_{ie}. The equivalent ciruit of the tank circuit is shown in the Figure 2.45.

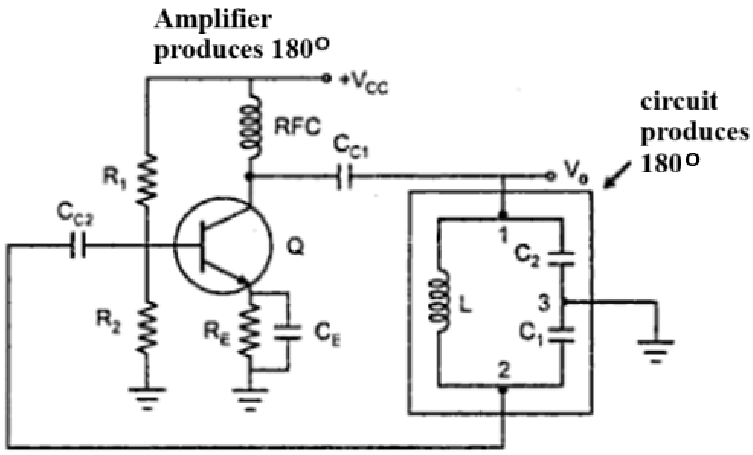


Figure 2.36: Colpitts oscillator

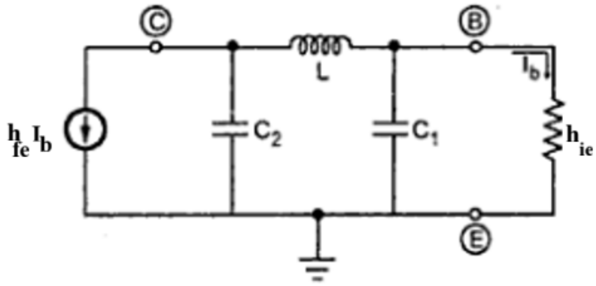


Figure 2.37: Equivalent colpitts oscillator

Converting the current source into the voltage source. We get the equivalent circuit as shown in the Figure 2.38.

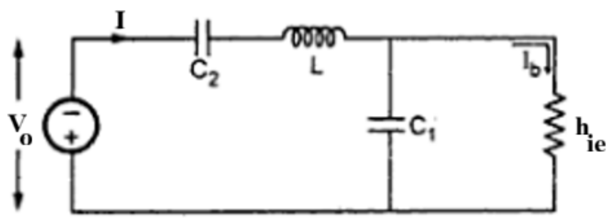


Figure 2.38: Simplified colpitts oscillator

$$V_o = h_{fe} I_b X_{C_2} = h_{fe} I_b \frac{1}{j\omega C_2} \tag{2.41}$$

The total current I, drawn from the supply is,

$$I = \frac{-V_o}{[X_{C_2} + X_L] + [X_{C_1} \| h_{ie}]} \tag{2.42}$$

Key point : The negative sign is because the current direction is assumed in the opposite direction to that, would be to polarities of V_o.

Now,

$$X_{C_2} + X_L = \frac{1}{j\omega C_2} + j\omega L$$

and,

$$X_{C_1} \| h_{ie} = \frac{\frac{1}{j\omega C_1} * h_{ie}}{\frac{1}{j\omega C_1} + h_{ie}}$$

Substituting in the equation 2.42,

$$I = \frac{-h_{fe} I_b (\frac{1}{j\omega C_2})}{[\frac{1}{j\omega C_2} + j\omega L] + [\frac{\frac{h_{ie}}{j\omega C_1}}{h_{ie} + \frac{1}{j\omega C_1}}]}$$

Replacing $j\omega$ by s,

$$\begin{aligned}
I &= \frac{-h_{fe} I_b (\frac{1}{sC_2})}{[\frac{1}{sC_2} + sL] + [\frac{\frac{h_{ie}}{sC_1}}{h_{ie} + \frac{1}{sC_1}}]} \\
I &= \frac{-h_{fe} I_b (\frac{1}{sC_2})}{\frac{1 + s^2 LC_2}{sC_2} + \frac{h_{ie}}{1 + sC_1 h_{ie}}} \\
I &= \frac{-h_{fe} I_b (\frac{1}{sC_2})(sC_2)(1 + sC_1 h_{ie})}{(1 + s^2 LC_2)(1 + sC_1 h_{ie}) + sC_2 h_{ie}} \\
I &= \frac{-h_{fe} I_b (1 + sC_1 h_{ie})}{s^3 LC_1 C_2 h_{ie} + s^2 LC_2 + sC_1 h_{ie} + 1 + sC_2 h_{ie}} \\
I &= \frac{-h_{fe} I_b (1 + sC_1 h_{ie})}{s^3 LC_1 C_2 h_{ie} + s^2 LC_2 + sh_{ie}(C_1 + C_2) + 1}
\end{aligned} \tag{2.43}$$

According to the current division in the parallel circuit,

$$\begin{aligned}
I_b &= I * \frac{X_{C_1}}{X_{C_1} + h_{ie}} = \frac{I * \frac{1}{j\omega C_1}}{h_{ie} + \frac{1}{j\omega C_1}} \\
I_b &= \frac{I}{(1 + sh_{ie} C_1)}
\end{aligned} \tag{2.44}$$

Substituting value of I from the equation 2.43 in equation 2.44 we get,

$$
\begin{aligned}
I_b &= \frac{-h_{fe}I_b(1+sC_1h_{ie})}{s^3LC_1C_2h_{ie}+s^2LC_2+sh_{ie}(C_1+C_2)+1} * \frac{1}{(1+sC_1h_{ie})} \\
I_b &= \frac{-h_{fe}I_b}{s^3LC_1C_2h_{ie}+s^2LC_2+sh_{ie}(C_1+C_2)+1} \\
1 &= \frac{-h_{fe}}{s^3LC_1C_2h_{ie}+s^2LC_2+sh_{ie}(C_1+C_2)+1}
\end{aligned} \tag{2.45}
$$

Replacing s by $j\omega$,

$$
\begin{aligned}
1 &= \frac{-h_{fe}}{-j\omega^3LC_1C_2h_{ie}-\omega^2LC_2+j\omega h_{ie}(C_1+C_2)+1} \\
1 &= \frac{-h_{fe}}{(1-\omega^2LC_2)+j\omega h_{ie}[C_1+C_2-\omega^2LC_1C_2]}
\end{aligned} \tag{2.46}
$$

It can be seen that, to satisfy the equation, the imaginary part of the denominator of the right hand side must be zero.

$$
\begin{aligned}
\omega h_{ie}[C_1+C_2-\omega^2LC_1C_2] &= 0 \\
C_1+C_2-\omega^2LC_1C_2 &= 0 \\
\omega^2 = \frac{C_1+C_2}{LC_1C_2} &= \frac{1}{L\frac{C_1C_2}{C_1+C_2}} \\
\omega &= \frac{1}{\sqrt{L\frac{C_1C_2}{C_1+C_2}}}
\end{aligned}
$$

Now, $\frac{C_1C_2}{C_1+C_2}$ is nothing but the equivalent of two capacitors C_1 and C_2 in series.

$$C_{eq} = \frac{C_1C_2}{C_1+C_2}$$

$$\omega = \frac{1}{\sqrt{LC_{eq}}} \tag{2.47}$$

$$f = \frac{1}{2\pi\sqrt{LC_{eq}}} \tag{2.48}$$

This is the frequency of the oscillations in the Colpitts oscillator. Substituting this frequency in the equation 2.46 and equation the magnitudes of the both sides, the restrinction on the value of h_{fe} can be obtained as,

$$h_{fe} = \frac{C_2}{C_1} \tag{2.49}$$

Key point : Thus the behaviour of Colpitts oscillator is similar to the Hartley oscillator, as basic LC oscillator circuit is same, except the tank circuit.

2.4 Analog Multipliers

The circuit which performs the multiplication of the two input voltages is called a multiplier circuit. Such multiplier circuits are used in the variety of applications such as squarer, square root extracter, frequency doubler etc.

2.4.1 Basic multiplier and its characteristics

A basic multiplier is an active circuit in which the output voltage is proportional to the product of the two input signals. A schematic symbol of such basic multiplier is shown in the Figure 2.39. The terminals V^+ and V^- are supply terminals for IC (Integrated Circuit) where dual supply is to

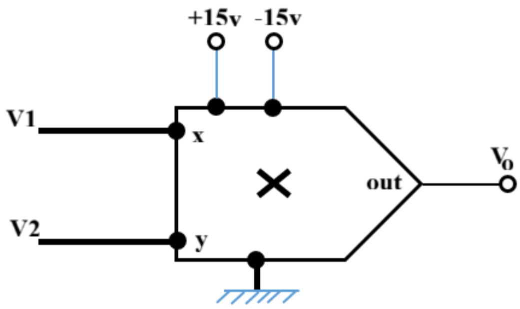


Figure 2.39: Multiplier symbol

be connected, generally $\mp 15V$ as show. The x and y are the two inputs V_1 and V_2 are connected. The output of such basic multiplier is,

$$V_o = KV_1V_2 \tag{2.50}$$

where K is constant and is equal to $1/V_{ref}$. Usually V_{ref} is set to $10V$ internally and hence,

$$V_o = \frac{V_1V_2}{10} \tag{2.51}$$

As long as it is ensured that both the input voltages are below the reference voltage,$V_1V_2 < V_{ref}$, the output of the basic multiplier will not saturate. Depending on the use of basic multiplier, it is necessary to restrict the polarity of one or both the inputs. Depending upon the polarity restriction, the operation is called as,

- **On quadrant multiplier:** In such operation, the polarities of both the inputs must always be positive.
- **Two quadrant multiplier:** A two quadrant multiplier functions properly if one input is held positive and the other is allowed to swing in both positive and negative.
- **Four quadrant multiplier:** If both the inputs are allowed to swing in both positive and negative directions, the operation is four quadrant multiplier operation.

2.4.2 Applications of multiplier

The multiplier is used in many practical applications. Some of these applications are,

- In communication it is used in amplitude modulation, phase modulation, frequency modulation, phase detection, suppressed carrier modulation etc.
- For voltage controlled attenuators and for voltage controlled amplifications.
- it is used for voltage divider, rectifier phase shift detection.
- It used for frequency converters, frequency doubling and frequency shifting etc.
- It used in oscillators to generate the waveforms and also used for square wave generation etc.

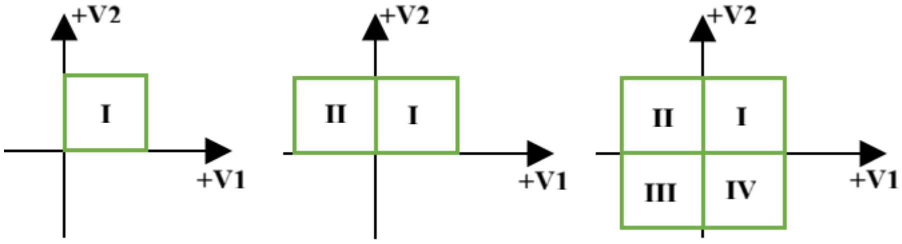


Figure 2.40: Multiplier modes of operation

Voltage divider using multiplier

The circuit in which output is the division of the two input signals, is called as a voltage divider. The use of multiplier as a voltage divider is shown in the Figure 2.41. The multiplier is used in the

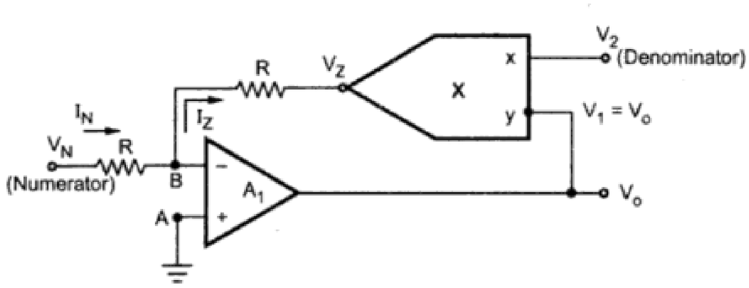


Figure 2.41: Voltage divider

feedback loop. The denominator is applied at the x input of the multiplier which is the voltage V_2. The numerator is applied at the input terminal of op-amp A_1. As node A is grounded, node B is also at virtual ground, hence $V_B = 0$. As op-amp input current is zero,

$$I_N = I_Z = \frac{V_N}{R} = -\frac{V_Z}{R} \tag{2.52}$$

Now,

$$V_Z = KV_1V_2 = KV_oV_2 \tag{2.53}$$

$$V_o = -\frac{V_N}{KV_2} \tag{2.54}$$

Thus the output is proportional to the division of the two input voltages V_N and V_2. The only requirement is that the input voltage V_2 must be negative. Hence divider circuits are best two quadrant circuit.

Squaring circuit using multiplier

The squaring circuit gives square of the input voltage applied. The multiplier inputs are connected together to get the squaring circuit as shown in Figure 2.42.

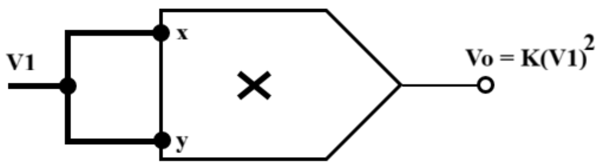


Figure 2.42: Squaring circuit

The same signal V_1 is appleid to both the input terminals of the multiplier.

$$V_o = KV_1V_1 = KV_1{}^2 \tag{2.55}$$

Thus the output is proportional to the square of the input.

Square Rooting circuit using multiplier

Similar to the squarting, the square rooting circuit can be obtained using multiplier. The circuit is shown in the Figure 2.43.

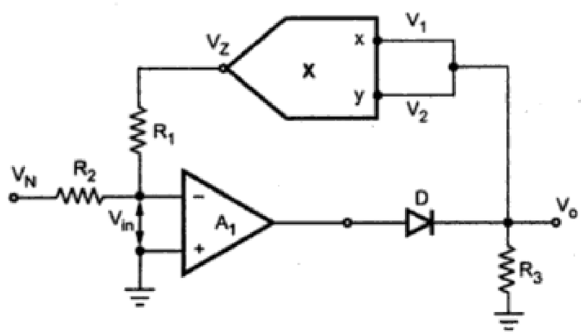


Figure 2.43: Square rooting circuit

A multiplier configured as squaring circuit is used in the feedback loop. The gain of the op-amp A_1 is say A and voltage between inverting and non-inverting terminal is V_{in}. So we can write,

$$V_o = -V_{in}A \Rightarrow V_{in} = -\frac{V_o}{A} \tag{2.56}$$

Now the voltage V_{in} is composed of two components, one that of V_Z and other that of V_N.

$$V_{in} = V_N\frac{R_1}{R_1+R_2} + V_Z\frac{R_2}{R_1+R_2} \tag{2.57}$$

But,

$$V_Z = KV_1V_2 = KV_0{}^2 \tag{2.58}$$

$$V_{in} = V_N\frac{R_1}{R_1+R_2} + KV_o{}^2\frac{R_2}{R_1+R_2} \tag{2.59}$$

Equating 2.56 and 2.59,

$$\begin{aligned} -\frac{V_o}{A} &= V_N\frac{R_1}{R_1+R_2} + KV_o{}^2\frac{R_2}{R_1+R_2} \\ V_o{}^2 &= -V_N\frac{R_1}{KR_2}[1+\frac{V_o(R_1+R_2)}{V_NAR_1}] \end{aligned} \tag{2.60}$$

Now A is high and $V_o < 0$ we can say,

$$\frac{V_o(R_1+R_2)}{V_N A R_1} \ll 1$$

$$\begin{aligned} {V_o}^2 &= -V_N \frac{R_1}{KR_2} \\ V_o &= \sqrt{-V_N \frac{R_1}{KR_2}} \end{aligned} \tag{2.61}$$

As seen from the equation 2.61, the output is proportional to the square root of V_N, but the V_N must be always negative, $V_N < 0$. Otherwise the circuit becomes latched up and normal operation can be restored by breaking the feedback loop. To avoid such problem, a series diode **D** is provided.

Frequency doubler using multiplier

The multiplication of two sine waves of same frequency but of possibly different amplitudes and phase, gives us a signal of a double frequency. Consider the two input signals as,

$$V_1 = V_{1m} sin(\omega t)$$

$$V_2 = V_{2m} sin(\omega t + \theta)$$

When the two inputs are given to a multiplier we get,

$$\begin{aligned} V_o &= KV_{1m} sin(\omega t).V_{2m} sin(\omega t+\theta) \\ V_o &= KV_{1m}V_{2m} sin(\omega t)[sin(\omega t)cos(\theta)+cos(\omega t)sin(\theta)] \\ V_o &= KV_{1m}V_{2m}[sin^2(\omega t)cos(\theta)+sin(\omega t)cos(\omega t)sin(\theta)] \end{aligned} \tag{2.62}$$

Now, $sin(\omega t)cos(\omega t) = \frac{sin(2\omega t)}{2}$.

$$\begin{aligned} V_o &= KV_{1m}V_{2m}[(\frac{1-cos2\omega t}{2})cos(\theta)+\frac{sin(2\omega t)}{2}sin(\theta)] \\ V_o &= \frac{KV_{1m}V_{2m}}{2}[cos\theta - cos\theta cos(2\omega t)+sin\theta sin(2\omega t)] \\ V_o &= \frac{KV_{1m}V_{2m}cos\theta}{2} - \frac{KV_{1m}V_{2m}}{2}cos(2\omega t-\theta)] \end{aligned} \tag{2.63}$$

The first term is DC for a phase difference of θ while the second term varies with time but at twice the frequency of the inputs. Thus circuit acts as a frequency doubler. Such a frequency doubler can be obtained by using a squaring circuit, as shown in the Figure 2.44. The two inputs are connected

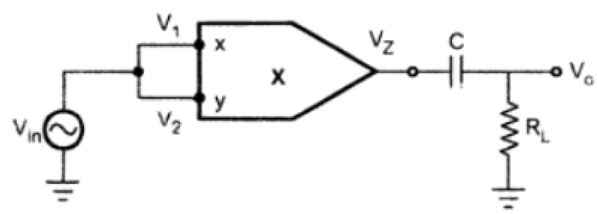


Figure 2.44: Squaring circuit as frequency doubler

together hence,

$$V_1 = V_2 = V_{in} = V_m sin(\omega t)$$

Hence $\theta = \theta^\circ$ which is phase difference between the two inputs.

$$\begin{aligned} V_Z &= K{V_{in}}^2 = KV_m sin(\omega t)^2 \\ V_Z &= K{V_m}^2 \frac{(1 - cos(2\omega t))}{2} \\ V_Z &= \frac{K{V_m}^2}{2} - \frac{K{V_m}^2}{2} cos(2\omega t) \end{aligned} \tag{2.64}$$

Thus the output of the multiplier is the DC. signal with time varying signal of double the input frequency. The capacitor C connected in series with the output blocks the DC and removes it. Thus we get,

$$V_Z = \frac{K{V_m}^2}{2} cos(2\omega t) \tag{2.65}$$

Thus the circuit acts as a frequency doubler.

2.5 Superheterodyne receiver

Among Armstrong's numerous inventions is the superheterodyne radio receiver. His earlier regenerative receiver, although sensitive, was prone to behave erratically (it frequently burst into oscillation). Tuned circuits, such as the parallel resonant circuit, are required to select a desired radio signal and to reject other signals. In addition, tuned circuits are used to enhance the gain of radio-frequency amplifiers. To tune a given circuit, its capacitance, inductance, or both must be changed. Several amplifiers, each with a tuned circuit, are often needed. The tuning of a radio thus required the simultaneous adjustment of several circuits a tuning knob was needed for each circuit.

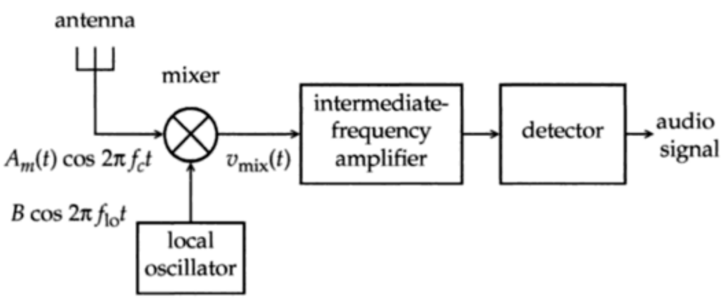


Figure 2.45: Superheterodyne receiver

Armstrong recognized that the carrier frequency of a signal could be changed through a nonlinear mixing process (Figure 2. 45). Consider the case for an amplitude-modulated signal

$$A_m(t)\cos(2\pi f_c t) \tag{2.66}$$

derived from an antenna system. A second high-frequency signal generated by the local oscillator of the receiver,

$$B\cos(2\pi f_{lo} t) \tag{2.67}$$

is also required. Suppose, initially, that the mixer results in an output voltage $v_{mix}(t)$ that is the product of its two inputs (a standard multiplier symbol is shown in Figure 2. 45), as expressed by

$$v_{mix}(t) = A_m(t)\cos(2\pi f_c t) \cdot B\cos(2\pi f_{lo} t)$$

$$= \frac{1}{2}A_m(t)B[\cos 2\pi(f_{lo} + f_c)t + \cos 2\pi(f_{lo} - f_c)t \tag{2.68}$$

The preceding result was obtained using the trigonometric identities for the cosine of the sum and difference of two angles as follows:

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \tag{2.69}$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \tag{2.70}$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha + \beta) + \cos(\alpha - \beta) \tag{2.71}$$

The output voltage of the mixer consists of two signals, one multiplied by

$$\cos 2\pi(f_{lo} + f_c)t \tag{2.72}$$

and the other multiplied by

$$\cos 2\pi(f_{lo} - f_c)t \tag{2.73}$$

These signals are two distinct amplitude-modulated signals, one having a carrier frequency of $f_{lo} + f_c$ and the other of $f_{lo} - f_c$. The amplitude of each is proportional to the amplitude of the original signal, that is, $A_m(t)$.

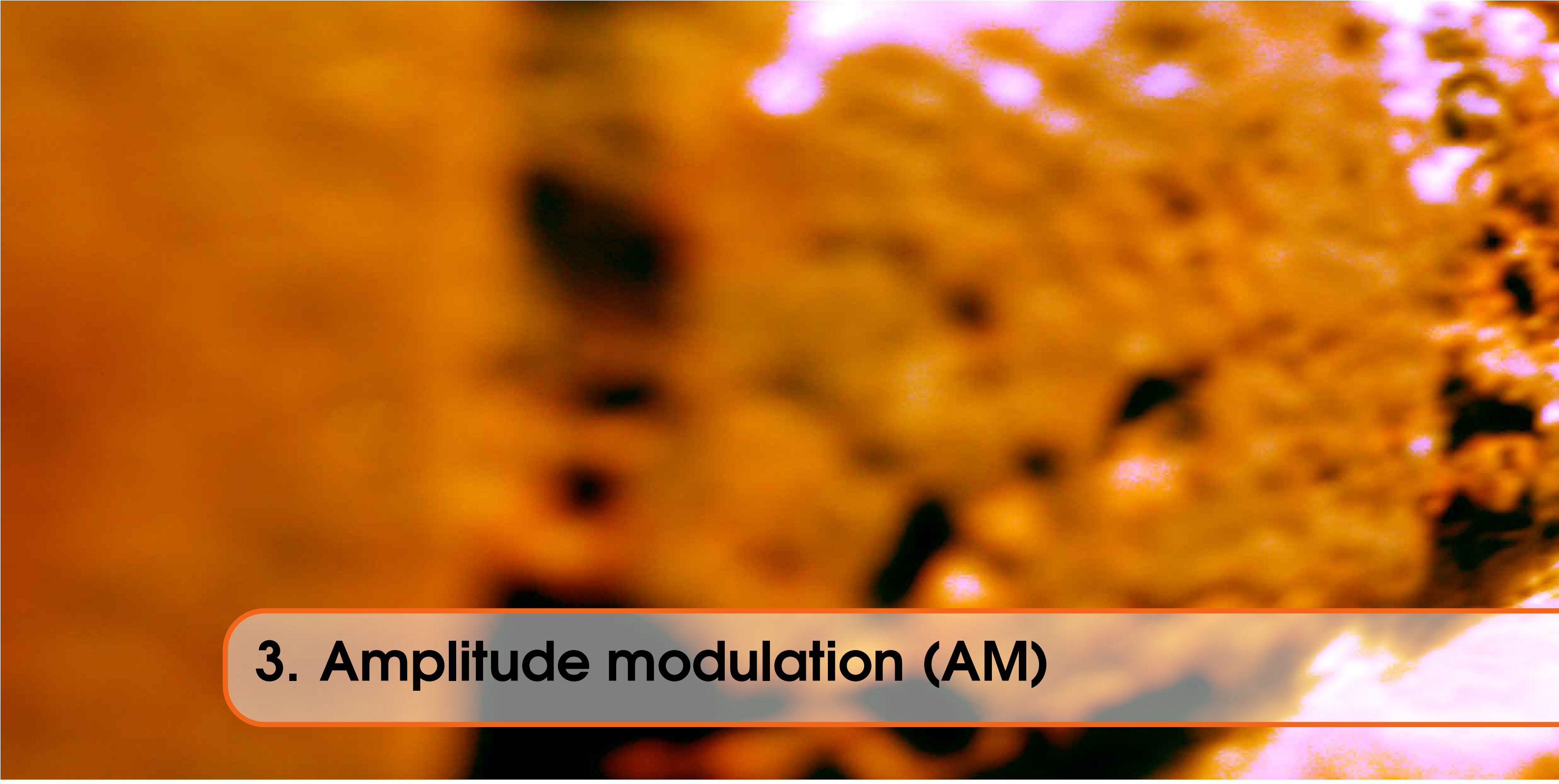
Consider the case for a typical AM broadcast receiver that might be tuned to receive an amplitude-modulated signal with a carrier frequency of 1350 kHz. Suppose that its local oscillator is generating an 1800 kHz signal. The output of the mixer would consist of two amplitude-modulated signals, one with a carrier frequency of 450 kHz and the other with a carrier frequency of 2250 kHz. If the intermediate-frequency amplifier is tuned to a frequency of 450 kHz, the component with a carrier frequency of 450 kHz would be amplified, whereas the 2250 kHz carrier signal would be lost. The 450 kHz signal would be detected after being amplified, thus yielding an audio output signal corresponding to the amplitude modulation $A_m(t)$ of the received signal. (A level shifting, generally achieved with a coupling capacitor, is also necessary to recover the audio signal.)

2.6 Conclusion

In this chapter, we explored the key analog functions that play a central role in communication systems. Analog filters were presented as indispensable components for frequency selection and noise suppression, ensuring signal quality and system performance. Oscillators, on the other hand, were shown to be the primary sources of periodic signals, with various topologies like Wien bridge, Hartley, and Colpitts demonstrating practical circuit design techniques.

The study of analog multipliers highlighted their importance in implementing modulation and signal processing functions that are fundamental to analog and mixed-signal communication systems.

Together, these three functions-filtering, oscillation, and multiplication-form the operational core of analog transmission. Understanding their principles and interactions equips students with the analytical and practical tools required for designing and optimizing more complex communication circuits encountered in later stages of telecommunication studies.



3. Amplitude modulation (AM)

3.1 Introduction

Modulation is a fundamental process in communication systems that allows information to be transmitted efficiently over long distances. It consists of varying certain properties of a high-frequency carrier signal-such as its amplitude, frequency, or phase-according to the instantaneous value of a lower-frequency information signal. This chapter focuses on one of the most widely used and historically significant forms of modulation: Amplitude Modulation (AM).

The chapter begins with a general introduction to modulation, explaining its necessity in overcoming transmission limitations such as antenna size, signal interference, and channel noise. It then explores the principles and mathematical representation of Amplitude Modulation, where the amplitude of a carrier wave is varied proportionally to the amplitude of the message signal.

Several forms of AM are presented in detail. The Double Sideband Suppressed Carrier (DSB-SC) system illustrates an efficient form of amplitude modulation that removes the carrier to conserve power and bandwidth. The corresponding demodulation techniques-used to recover the original message-are discussed to highlight practical circuit implementations.

Next, the Conventional AM or Double Sideband with Carrier (DSB-AM) system is analyzed, followed by its demodulation methods, which are simpler and widely used in commercial broadcasting. The chapter concludes with the study of Single Sideband (SSB) modulation, a more bandwidth-efficient form that transmits only one sideband of the modulated signal, along with its associated demodulation techniques.

Through this chapter, students will gain both theoretical and practical insight into the operation, analysis, and applications of amplitude modulation systems, which continue to serve as the foundation for many modern communication technologies.

3.2 Amplitude modulation (AM)

In amplitude modulation, the message $m(t)$ is impressed on the amplitude of the carrier signal $c(t)$. There are several different ways of amplitude modulating the carrier signal by $m(t)$, each of which results in different spectral characteristics for the transmitted signal. Next, we describe these methods, which are called : double sideband, suppresed carrier AM, conventional double-sideband AM, single sideband AM.

3.2.1 Double sideband Suppressed Carrier AM

A double sideband, suppressed carrier(DSB SC) signal is obtained by multiplyin the message signal $m(t)$ with the carrier signal $c(t)$. Thus, we have the amplitude mosulated signal,

$$u(t) = m(t)c(t) = m(t)A_c cos(2\pi f_c t + \phi_c)$$

Spectrum of the modulated signal

The spectrum can be obtained by taking the Fourier transform of $u(t)$.

$$U(f) = \mathscr{F}[m(t)] \star \mathscr{F}[A_c cos(2\pi f_c t + \phi_c)]$$
$$U(f) = M(f) \star \frac{A_c}{2}[e^{j\phi_c}\delta(f - f_c) + e^{-j\phi_c}\delta(f + f_c)]$$
$$U(f) = \frac{A_c}{2}[e^{j\phi_c}M(f - f_c) + e^{-j\phi_c}M(f + f_c)]$$

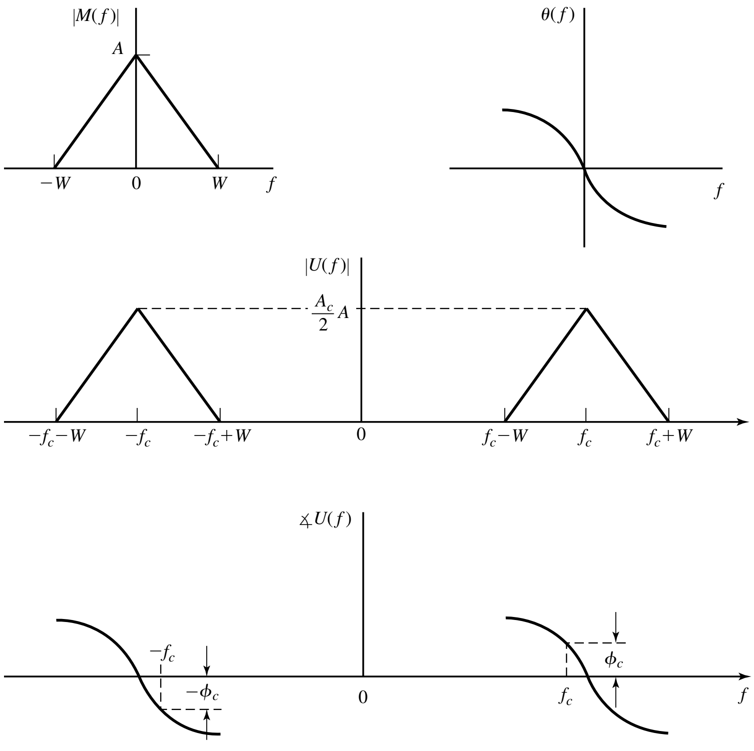


Figure 3.1: Magnitude and phase spectra for M(f) and U(f)

.

We observe that the magnitude of the spectrum of the message signal $m(t)$ has been translated in frequency by an amount f_c. The phase of the message signal has been translated in frequency and offset by the carrier phase ϕ_c. Furthermore, the bandwidth occupancy of the amplitude modulated signal is $2W$, whereas the bandwidth of the modulated signal $m(t)$ is W. Therefore, the channel bandwidth required to transmit the modulated signal $u(t)$. is $B_c = 2W$. The frequency content of the modulated signal $u(t)$ in the frequency band $|f| > f_c$ is called the upper sideband of $U(f)$, and the frequency content in the frequency band $|f| < f_c$ is called the lower sideband of $U(f)$. It is important to note that either one of the sidebands of the $U(f)$ contains all the frequencies that are in $M(f)$.

The other characteristic of the modulated signal $u(t)$ is that it does not contain a carrier component. That is, all the transmitted power is contained in the modulating (message) signal $m(t)$.This is evident from observing the spectrum of $U(f)$. We note that, as long as $m(t)$ does not have any DC component, there is no impulse in $U(f)$ at $f = f_c$, which would be the case if a carrier component was contained in the modulated signal $u(t)$. For this reason, $u(t)$ is called a suppressed carrier signal. Therefore, $u(t)$ is a DSB-SC AM signal.

Exercise 3.1 Suppose that the modulating signal $m(t)$ is a sinusoid of the form. Determine the DSB-SC AM signal and its upper and lower sidebands.

$$m(t) = a\cos(2\pi f_m t) \qquad f_m \ll f_c$$

Solution

The DSB-SC AM is expressed in the time domain as,

$$u(t) = m(t)c(t) = A_c a \cos(2\pi f_m t)\cos(2\pi f_c t + \phi_c)$$
$$u(t) = \frac{A_c a}{2}\cos[2\pi(f_c - f_m)t + \phi_c)] + \frac{A_c a}{2}\cos[2\pi(f_c + f_m)t + \phi_c)]$$

In the frequency domain, the modulated signal has the form,

$$U(f) = \frac{A_c a}{2}[e^{j\phi_c}\delta(f - f_c + f_m) + e^{-j\phi_c}\delta(f + f_c - f_m)]$$
$$+\frac{A_c a}{2}[e^{j\phi_c}\delta(f - f_c - f_m) + e^{-j\phi_c}\delta(f + f_c + f_m)]$$

This spectrum is shown in Figure 3.2

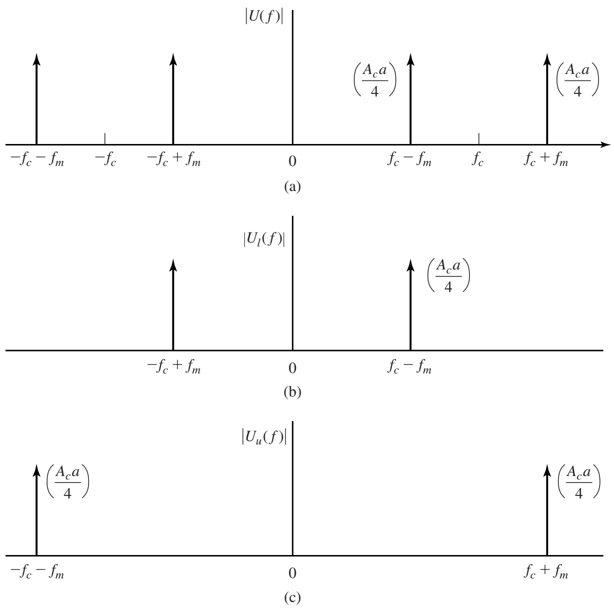


Figure 3.2: (a) Magnitude for a sinusoidal message iagnal and (b) its lower and (c) upper sidebans
.

The lower sideband of $u(t)$ is signal,

$$u_l(t) = \frac{A_c a}{2}\cos[2\pi(f_c - f_m)t + \phi_c)]$$

and its spectrum is illustrated in Figure 3.2(b). Finally, the upper sideband of $u(t)$ is the signal,

$$u_u(t) = \frac{A_c a}{2}\cos[2\pi(f_c + f_m)t + \phi_c)]$$

and its spectrum is illustrated in Figure 3.2(c).

Power content of DSB-SC signals

In order to compute the power content of the DSB-SC signal, without loss of generality we can assume that the phase of the signal is set to zero. This is because the power in a signal is independent of the phase of the signal. The time average autocorrelation function of the signal $u(t)$ is given by,

$$\begin{aligned}
R_u(\tau) &= \lim_{T\to\infty}\frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} u(t)u(t-\tau)dt \\
R_u(\tau) &= \lim_{T\to\infty}\frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} A_c^2 m(t)m(t-\tau)\cos(2\pi f_c t)\cos(2\pi f_c(t-\tau))dt \\
R_u(\tau) &= \frac{{A_c}^2}{2}\lim_{T\to\infty}\frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} m(t)m(t-\tau)[\cos(4\pi f_c t - 2\pi f_c\tau) + \cos(2\pi f_c\tau)]dt \\
R_u(\tau) &= \frac{{A_c}^2}{2}R_m(\tau)\cos(2\pi f_c\tau)
\end{aligned} \tag{3.1}$$

where we have used the fact that,

$$\lim_{T\to\infty}\int_{\frac{-T}{2}}^{\frac{T}{2}} m(t)m(t-\tau)\cos(4\pi f_c t - 2\pi f_c\tau)dt = 0$$

This is because,

$$\begin{aligned}
&\int_{-\infty}^{\infty} m(t)m(t-\tau)\cos(4\pi f_c t - 2\pi f_c\tau)dt \\
&\overset{(a)}{=} \int_{-\infty}^{\infty} \mathscr{F}[m(t-\tau)][\mathscr{F}(m(t)\cos(4\pi f_c t - 2\pi f_c\tau))]^* df \\
&= \int_{-\infty}^{\infty} e^{-j2\pi f\tau} M(f)\left[\frac{M(f-2f_c)e^{-j2\pi f_c\tau}}{2} + \frac{M(f+2f_c)e^{j2\pi f_c\tau}}{2}\right]^* df \overset{(b)}{=} 0
\end{aligned}$$

where in (a) we have used Parseval's relation and (b) we have used the fact taht $M(f)$ is limited to the frequency band $[-W,W]$ and $W \ll f_c$, hence, there is no frequency overlap between $M(f)$ and $M(f \pm 2f_c)$. By taking the Fourier transform of both sides of equation 3.1, we can obtain the power-spectral density of the modulated signal as,

$$\begin{aligned}
S_u(f) &= \mathscr{F}\left[\frac{{A_c}^2}{2}R_m(\tau)\cos(2\pi f_c\tau)\right] \\
S_u(f) &= \frac{{A_c}^2}{4}[S_m(f-f_c) + S_m(f+f_c)]
\end{aligned} \tag{3.2}$$

This shows that the power spectraldensity of the DSB SC signal is the power spectral density of the message shifted upward and downward by f_c and scaled by ${A_c}^2/4$. To obtain the total power in the modulated signal, we can either substitute $\tau = 0$ in the time-average autocorrelation function as given in equation 3.1, or we can integrate the power spectral density of the modulated signal given in equation 3.2 over all frequencies. Using the first approach from equation 3.1, we have,

$$P_u \stackrel{\tau=0}{=} \frac{A_c{}^2}{2} R_m(0) = \frac{A_c{}^2}{2} P_m \tag{3.3}$$

where $P_m = R_m(0)$ is the power in the message signal.

Exercise 3.2 In Example 3.1, determine the power spectral density of the modulated signal, the power in the modulated signal, and the power in each of the sidebands. ■

Solution

The message signal is $m(t) = a\cos 2\pi f_m t$, its power spectral density is given by,

$$S_m(f) = \frac{a^2}{4}\delta(f - f_m) + \frac{a^2}{4}\delta(f + f_m)] \tag{3.4}$$

Substituting in equation 3.2 we obtain the power spectral density of the modulated signal as,

$$S_u(f) = \frac{A_c{}^2 a^2}{16}[\delta(f - f_m - f_c) + \delta(f + f_m - f_c) + \delta(f - f_m + f_c) + \delta(f + f_m + f_c)]$$

The total power in the modulated signal is the integral of $S_u(f)$ and is given by,

$$P_u = \int_{-\infty}^{\infty} S_u(f)df = \frac{A_c{}^2 a^2}{4}$$

Because of symmetry of sidebandes the powers the upper and lower sidebands, P_{us}, and P_{ls}, are equal and given by,

$$P_{ls} = P_{us} = \frac{A_c{}^2 a^2}{8}$$

It can also be observed from the power spectral density of the DSB SC signal (see equation 3.2) that the bandwidth of the modulated signal is $2W$, twice the bandwidth of the message signal, and that there exists no impulse at the carrier frequency $\pm f_c$ in the power spectral density. Therfore, the modulated signal is suppressed carrier (SC).

3.2.2 Demodulation of DSB-SC AM signals

In the absence of noise, and with the assummption of an ideal channel, the received signal is equal to the modulated signal,

$$r(t) = u(t) = A_c m(t)\cos(2\pi f_c t + \phi_c) \tag{3.5}$$

Suppose we demodulate the received signal by first multiplying r(t) by a locally generated sinusoid $\cos(2\pi f_f t + \phi_c)$, where ϕ_c is the phase of the sinusoid, and then passing the product signal through an ideal lowpass filter having a bandwidth W. The multiplication of $r(t)$ with $\cos(2\pi f_f t + \phi_c)$ yields

$$r(t)\cos(2\pi f_c t + \phi) = A_c m(t)\cos(2\pi f_c t + \phi_c)\cos(2\pi f_c t + \phi)$$
$$r(t)\cos(2\pi f_c t + \phi) = \frac{1}{2}A_c m(t)\cos(\phi_c - \phi) + \frac{1}{2}A_c m(t)\cos(4\pi f_c t + \phi + \phi_c)$$

The lowpass filter rejects the double frequency components and passes only the lowpass components. Hence, its output is,

$$y_l(t) = \frac{1}{2} A_c m(t) \cos(\phi_c - \phi) \tag{3.6}$$

Note that $m(t)$ is multiplied by $\cos(\phi_c - \phi)$. Thus, the desired signal is scaled in amplitude by a factor that depends on the phase diffrence between the phase ϕ_c of the carrier in the received signal and the phase ϕ of locally generated sinusoid. When $\phi_c \neq \phi$, the amplitude of the desired signal is redused by the factor $\cos(\phi_c - \phi)$. If $\phi_c - \phi = 45°$, the amplitude of the desired signal is reduced by $\sqrt{2}$ and the signal power is reduced by a factor of two. If $\phi_c - \phi = 90°$, the desired signal component vanishes. The above discussion demonstrates the need for a phase coherent or synchronous demodulator for recovering the message signal $m(t)$ from the received signal. That is the phase ϕ of the locally generated sinusoid should ideally be equal to the phase ϕ_c of the received carrier signal. A sinusoid that is phase locked to the phase of the received carrier can be generated at the receiver in one of two ways. One method is to add a carrier component into the transmitted signal as illustrated in Figure 3.3. We call such a carrier component "a pilot tone" Its amplitude

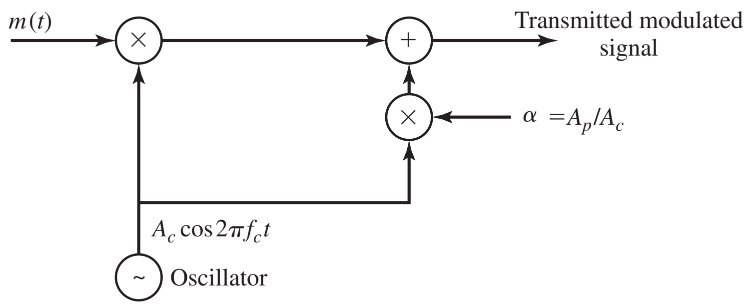


Figure 3.3: Addition of a pilot tone to a DSB AM signal

A_p and , hence its power $A_p{}^2/2$ is selected to be significantly smaller than that of the modulated signal $u(t)$. Thus the transmitted signal is double sideband, but it is no longer a suppressed carrier signal. At the receiver, a narrowband filter tuned to frequency f_c is used to filter out the pilot signal component end its output is used to multiply the received signal as shown in Figure 3.4. We can show that the presence of the pilot signal results in a DC component in the demodulated

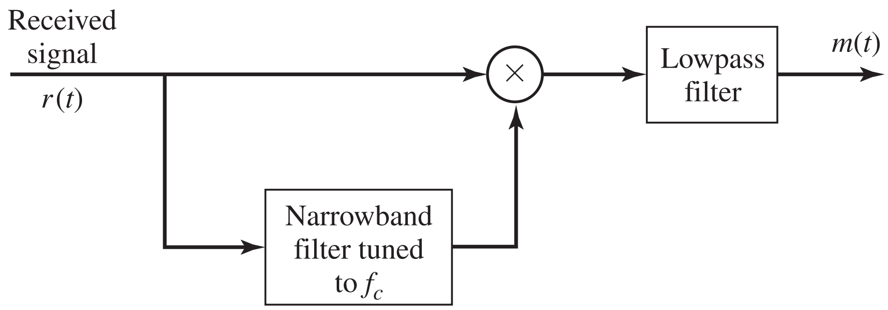


Figure 3.4: Use of a pilot tone to demodulate a DSB AM signal

signal, which must be subtracted out in order to recover $m(t)$. The addition of the pilot tone to the transmitted signal has the disadvantage of requiring that a certain portion of the transmitted signal power must be allocated to the transmission of the pilot. As an alternative , we may generate a phase-locked sinusoidal carrier from the received signal $r(t)$ without the need of the pilot signal. This can be accomplished by use of a phase-locked loop (PLL).

3.2.3 Conventional amplitude modulation

Aconventional AM signal consists of a large carrier component in addition to the double sideband AM modulated signal. The transmitted siganl is expressed as,

$$u(t) = A_c[1 + m(t)]\cos(2\pi f_c t + \phi_c) \tag{3.7}$$

where the message waveform is constrained to satisfy the condition that $|m(t)| \leq 1$. We observe that $A_c m(t)\cos(2\pi f_c t + \phi_c)$ is a double sideband AM signal and $A_c \cos(2\pi f_c t + \phi_c)$ is the carrier component. Figure 3.5 illustrates an AM signal in the time domain. If $|m(t)| \leq 1$, the amplitude

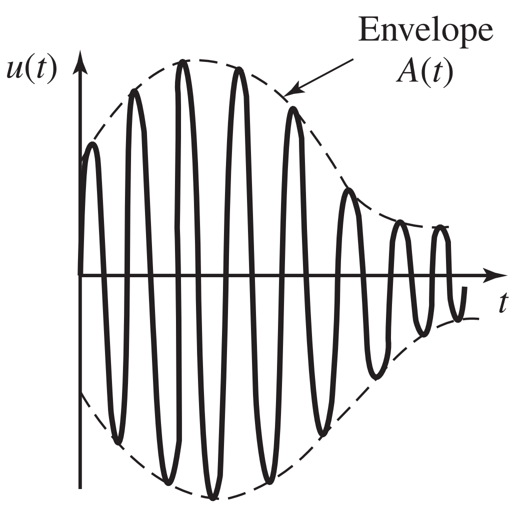


Figure 3.5: Conventional AM signal in the time domain

$A_c[1 + m(t)]$ is always positive. This is the desired condition for conventional DSB AM that makes it easy to demodulate. On the hand, if $m(t) < -1$ for some t, the AM signal is said to be overmodulated and its demodulation is rendered more complex. In practice, m(t) is scaled so that its magnitude is always less than unity. It is sometimes convenient to express $m(t)$ as,

$$m(t) = am_n(t)$$

where $m_n(t)$ is normalized such that its minimum value is -1. This can be done, for example, by defining,

$$m_n(t) = \frac{m(t)}{max|m(t)|}$$

The scale factor a is called the **modulation index**. Then the modulated signal can be expressed as,

$$u(t) = A_c[1 + am_n(t)]\cos(2\pi f_c t) \tag{3.8}$$

Bandwidth of conventional AM signal

If $m(t)$ is a deterministic signal with Fourier transform $M(f)$, the spectrum of the amplitude modulated signal $u(t)$ is,

$$
\begin{aligned}
U(f) &= \mathscr{F}[am_n(t)] \star \mathscr{F}[A_c \cos(2\pi f_c + \phi_c)] + \mathscr{F}[A_c \cos(2\pi f_c + \phi_c)] \\
U(f) &= aM_n(f) \star \frac{A_c}{2}[e^{j\phi_c}\delta(f - f_c) + e^{-j\phi_c}\delta(f + f_c)] \\
&\quad + \frac{A_c}{2}[e^{j\phi_c}\delta(f - f_c) + e^{-j\phi_c}\delta(f + f_c)] \\
U(f) &= \frac{A_c}{2}[e^{j\phi_c} aM_n(f - f_c) + e^{j\phi_c}\delta(f - f_c) \\
&\quad + e^{-j\phi_c} aM_n(f + f_c) + e^{-j\phi_c}\delta(f + f_c)]
\end{aligned}
$$

Obviously, the spectrum of a conventional AM signal occupies a bandwidth **twice the bandwidth** of the message signal.

Exercise 3.3 Suppose that the modulating signal $m_n(t)$ is a sinusoid of the form,

$$m_n(t) = \cos 2\pi f_m t \qquad f_m \ll f_c$$

Determine the DSB AM signal, its upper and lower sidebandes, and its spectrum, assuming a modulation index of a. ■

Solution

From equation 3.7, the DSB AM signal is expresssed as,

$$
\begin{aligned}
u(t) &= A_c[1 + a\cos 2\pi f_m t]\cos(2\pi f_c t + \phi_c) \\
u(t) &= A_c \cos(2\pi f_c t + \phi_c) + \frac{A_c a}{2}\cos[2\pi(f_c - f_m)t + \phi_c] \\
&\quad + \frac{A_c a}{2}\cos[2\pi(f_c + f_m)t + \phi_c]
\end{aligned}
$$

The lower sideband component is,

$$u_l(t) = \frac{A_c a}{2}\cos[2\pi(f_c - f_m)t + \phi_c]$$

The upper sideband component is,

$$u_u(t) = \frac{A_c a}{2}\cos[2\pi(f_c + f_m)t + \phi_c]$$

The spectrum of the DSB AM signal $u(t)$ is,

$$
\begin{aligned}
U(f) &= \frac{A_c}{2}[e^{j\phi_c}\delta(f - f_c) + e^{-j\phi_c}\delta(f + f_c)] \\
&\quad + \frac{A_c a}{4}[e^{j\phi_c}\delta(f - f_c + f_m) + e^{-j\phi_c}\delta(f + f_c - f_m)] \\
&\quad + \frac{A_c a}{4}[e^{j\phi_c}\delta(f - f_c - f_m) + e^{-j\phi_c}\delta(f + f_c + f_m)]
\end{aligned}
$$

The spectrum $|U(f)|$ is shown in Figure 3.6.

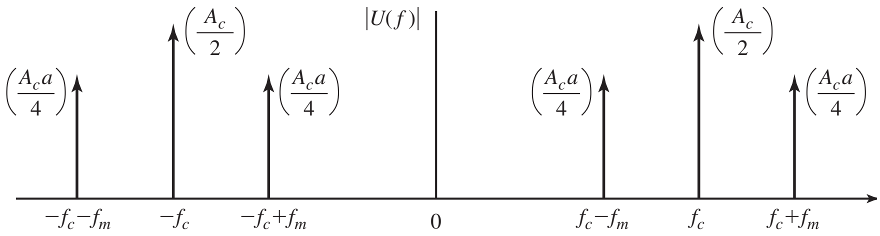


Figure 3.6: Spectrum of a DSB AM signal

It is interesting to note that the power of the carrier component, which is ${A_c}^2/2$, exceeds the total power ${A_c}^2 a/2$ of the two sidebands.

Power for conventional AM signal

Conventional AM signal is similar to DSB when $m(t)$ is substituted with $1 + am_n t$. As we have already seen in the DSB SC case, the power in the modulated signal is,

$$P_u = \frac{{A_c}^2}{2} P_m$$

where P_m denotes the power in the message signal. For the conventional AM.

$$P_m = \lim_{T\to\infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} (1 + am_n(t))^2 dt$$
$$P_m = \lim_{T\to\infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} (1 + a^2 {m_n}^2(t)) dt$$

where we have assumed that the average of $m_n(t)$ is to zero. This is a reasonable assumption for many signals including audio signals. Therefore, for conventional AM,

$$P_m = 1 + a^2 P_{m_n}$$

and hence,

$$P_u = \frac{{A_c}^2}{2} + \frac{{A_c}^2}{2} a^2 P_{m_n}$$

The first component is due to the existence of the carrier and this component does not carry any information. The second component is the information carrying component. Note that the second component is usually much smaller than the first component. This shows that the conventional AM systems are far less power efficient compared with DSB SC systems. the advantage of conventional AM is that it is easily demodulated.

3.2.4 Demodulation of conventional DSB AM

The major advantage of conventional AM signal transmission is the ease with wich the signal can be demodulated. There is no need for a synchronous demodulator. Since the message signal $m(t)$ satisfies the condition $|m(t)| < 1$, the envelope (amplitude) $1 + m(t) > 1$. If we rectify the received signal, we eleminate the negative values without affecting the massege signal as shown in Figure 3.7. The rectified signal is equal to $u(t)$ when $u(t) > 0$ and zero when $u(t) < 0$. The message signal is recovered by passing the rectified signal through a lowpass filter whose bandwidth matches that of the message signal. The combination of the rectifier and the lowpass filter is called an **envelope**

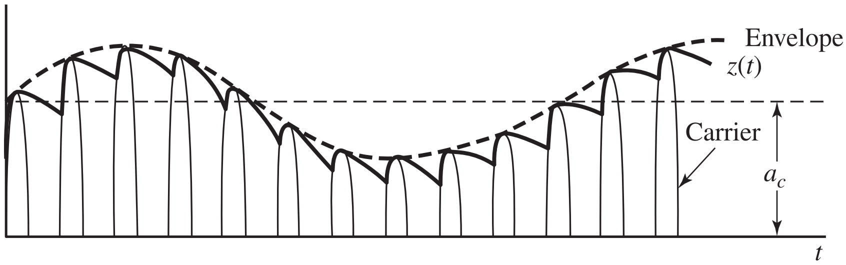


Figure 3.7: Envelope detection of conventional AM signal

detector.
Ideally, the output of the envelope detector is of the form,

$$d(t) = g_1 + g_2 m(t)$$

where g_1 represents a DC component and g_2 is a gain factor due to the signal demodulator. The DC component can be eliminated by passing $d(t)$ through a transformer, whose output is $g_2 m(t)$. The simplicity of the demodulator has made conventional DSB AM a practical choice for AM radio broadcasting. Since there are literally billions of radio receivers, an inexpensive implementation of the demodulator is extremely important. The powe inefficiency of conventional AM is justified by the fact that there are few broadcast transmitters relative to the number of receivers. Consequently. it is cost effective to construct powerful transmitters and sacrifice power efficiency in order to simplify the signal demodulation at the receivers.

3.2.5 Single sideband AM

A DSB SC AM signal required a channel bandwidth of $B_c = 2W$ Hz for transmission, where W is the bandwidth of the baseband signal. However. the two sidebandes are redundant. Next we demonstrate that the transmission of either sideband is sufficient to reconstruct the message signal $m(t)$ at the receiver. Thus, we reduce the bandwidth of the transmitted to that of the baseband signal. First, we demonstrate that a single sideband (SSB) AM signal is represented mathematically as,

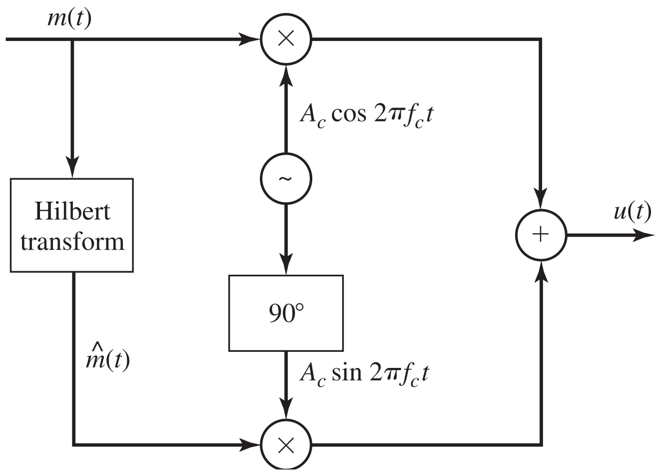


Figure 3.8: Generation of a SSB AM signal

$$u(t) = A_c m(t)\cos(2\pi f_c t) \mp A_c \hat{m}(t)\sin(2\pi f_c t) \tag{3.9}$$

where $\hat{m}(t)$ is the Hilbert transform of $m(t)$, the plus ans minus determines which sideband we obtain. We recall that the Hilbert transform may be viewed as a linear filter with impulse response $h(t) = 1/\pi t$ and frequancy response,

$$H(f) = \begin{cases} -j, & f > 0 \\ j, & f < 0 \\ 0, & f = 0 \end{cases} \tag{3.10}$$

Therefore, the SSB AM signal $u(t)$ may be generated by using the system configuration shown in Figure 3.8. The method shown in Figure 3.8 for generating a SSB AM signal is one that employs a Hilbert transform filter. Another method, illustrated in Figure 3.9, generats a DSB SC AM signal and then employs a filter which selects either the upper sideband or the lower sideband of the double sideband AM signal.

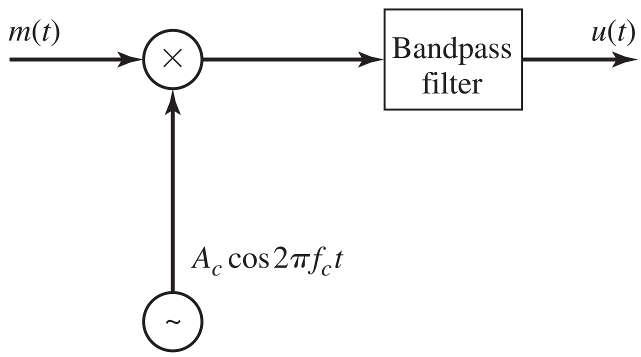


Figure 3.9: Generation of a single sideband AM signal by filtering on of the sidebandes of a DSB SC AM signal

Spectral characteristics of the signal sideband signal

Let $m(t)$ be a signal with Fourier transform $M(f)$. An upper single sideband amplitude modulated signal USSB AM is obtained by eliminating the lower sideband of a DSB amplitude modulated signal. Suppose we eliminate the lower sideband of the DSB AM signal, $u_{DSB}(t) = 2A_c m(t)\cos(2\pi f_c t)$, by passing it through a highpass filter whose transfer function is given by,

Obviously H(f) can be written as,

$$H(f) = u_{-1}(f - f_c) + u_{-1}(-f - f_c)$$

where $u_{-1}(.)$ represents the unit step function. Therefore the spectrum of the USSB AM signal is given by,

$$U_u(f) = A_c M(f - f_c)u_{-1}(f - f_c) + A_c M(f + f_c)u_{-1}(-f - f_c)$$

or equivalently,

$$U_u(f) = A_c M(f)u_{-1}(f)|_{f=f-f_c} + A_c M(f)u_{-1}(-f)|_{f=f+f_c} \tag{3.11}$$

Taking inverse Fourier transform of both sides of equation 3.11 and using the modulation property of the Fourier transform, we obtain,

$$u_u(t) = A_c m(t) \star \mathscr{F}^{-1}[u_{-1}(f)]e^{j2\pi f_c t} + A_c m(t) \star \mathscr{F}^{-1}[u_{-1}(-f)]e^{-j2\pi f_c t} \tag{3.12}$$

By noting that,

$$\mathscr{F}\left[\frac{1}{2}\delta(t)+\frac{j}{2\pi t}\right]=u_{-1}(f) \quad , \quad \mathscr{F}\left[\frac{1}{2}\delta(t)-\frac{j}{2\pi t}\right]=u_{-1}(-f) \tag{3.13}$$

and substituting equation 3.13 in equation 3.12 we obtain,

$$\begin{aligned} u_u(t) &= A_c m(t)\star\left[\frac{1}{2}\delta(t)+\frac{j}{2\pi t}\right]e^{j2\pi f_c t}+A_c m(t)\star\left[\frac{1}{2}\delta(t)-\frac{j}{2\pi t}\right]e^{-j2\pi f_c t} \\ &= \frac{A_c}{2}[m(t)+j\hat{m}(t)]e^{j2\pi f_c t}+\frac{A_c}{2}[m(t)-j\hat{m}(t)]e^{-j2\pi f_c t} \end{aligned} \tag{3.14}$$

where we have used the identities,

$$m(t)\star\delta(t)=m(t)$$
$$m(t)\star\frac{1}{\pi t}=\hat{m}(t)$$

Using Euler's relations in equation 3.14, we obtain,

$$u_u(t)=A_c m(t)\cos(2\pi f_c t)-A_c\hat{m}(t)\sin(2\pi f_c t) \tag{3.15}$$

which is the time domain representation USSB AM signal. The expression for the LSSB AM signal can be derived by noting that,

$$u_u(t)+u_l(t)=u_{DSB}(t)$$

or,

$$A_c m(t)\cos(2\pi f_c t)-A_c\hat{m}(t)\sin(2\pi f_c t)+u_l(t)=2A_c m(t)\cos(2\pi f_c t)$$

and therefore,

$$u_l(t)=A_c m(t)\cos(2\pi f_c t)+A_c\hat{m}(t)\sin(2\pi f_c t) \tag{3.16}$$

Thus, the time domain representation of a SSB AM signal can in general be expressed as,

$$u_{SSB}(t)=A_c m(t)\cos(2\pi f_c t)\mp A_c\hat{m}(t)\sin(2\pi f_c t) \tag{3.17}$$

where the minus sign corresponds to the USSB AM signal and the plus sign corresponds to the LSSB AM signal.

Exercise 3.4 Suppose that the modulating signal is a sinusoid of the form,

$$m(t)=\cos(2\pi f_m t), \quad f_m \ll f_c$$

Determine the two possible SSB AM signals. ■

Solution

The Hilbert transform of m(t) is,

$$\hat{m}(t)=\sin(2\pi f_m t) \tag{3.18}$$

hence,

$$u(t)=A_c\cos(2\pi f_m t)\cos(2\pi f_c t)\mp A_c\sin(2\pi f_m t)\sin(2\pi f_c t) \tag{3.19}$$

If we take the upper $(-)$ sign we obtain the upper sideband signal,

$$u_u(t) = A_c \cos 2\pi(f_c + f_m)t$$

On the other hand, if we take the lower $(+)$ sign in equation 3.19 we obtain the lower sideband signal.

$$u_l(t) = A_c \cos 2\pi(f_c - f_m)t$$

The spectra of $u_l(t)$ and $u_u(t)$ were previously given in Figure 3.2.

3.2.6 Demodulation of SSB AM signals

To recover the message signal $m(t)$ in the received SSB AM signal, we require a phase coherent or synchronous demodulator, as was the case for DSB SC AM signals. Thus, for the USSB signal as given in equation 3.17, we have.

$$\begin{aligned} r(t)\cos(2\pi f_c t) &= u(t)\cos(2\pi f_c t + \phi) \\ &= \frac{1}{2}A_c m(t)\cos\phi + \frac{1}{2}A_c \hat{m}(t)\sin\phi + C \end{aligned} \tag{3.20}$$

where C is a double frequency terms. By passing the product signal in equation 3.20 through an ideal lowpass filter, the double frequancy components are eliminated, leaving us with,

$$y_l(t) = \frac{1}{2}A_c m(t)\cos\phi + \frac{1}{2}A_c \hat{m}(t)\sin\phi \tag{3.21}$$

Note that the effect of the phase offset is not onlyto reduce the amplitude of the desired signal $m(t)$ by $\cos\phi$, but it also results in an undesirable sideband signal due to the presence of $\hat{m}(t)$ in $y_l(t)$. The latter component was not present in a DSB SC signal and hence, it was not a factor. However, it is an important element that contributes to the distortion of the demodulated SSB signal. The transmission of a pilot tone at the carrier frequency is a very effective method for providing a phase coherent reference signal for performing synchronous demodulation at the receiver. Thus, the undesirable sideband signal component is eliminated. However, this means that a portion of the transmitted power must be allocated to the transmission of the carrier. The spectral efficiency of SSB AM makes this modulation method very attractive for use in voice communications over telephone channels. In this application, a pilot tone is transmitted for synchronous demodulation and shared among several channels. The filter method shown in Figure 3.9 for selecting one of the two signal sidebands for transmission is particulary difficult to implement when the message signal $m(t)$ has a large power concentrated in the vicinity of $f = 0$. In such a case, the sideband filter must have an extremely sharp cutoff in the vicinity of the carrier in order to reject the second sideband. Such filter characteristics are very difficult to implement in practice.

3.3 Power in SSB signal

The total transmitted power in amplitude modulation can be expressed as:

$$P_t = P_c\left(1 + \frac{m^2}{2}\right) \tag{3.22}$$

where:

- P_t is the total transmitted power,
- P_c is the carrier power,

- m is the modulation index.

The total power is therefore the sum of the carrier power and the sideband power:

$$P_t = P_c + P_s \tag{3.23}$$

Hence, the total sideband power is:

$$P_s = P_c \frac{m^2}{2} \tag{3.24}$$

In conventional AM transmission, this sideband power is equally divided between the upper and lower sidebands:

$$P_{\text{upper}} = P_{\text{lower}} = P_c \frac{m^2}{4} \tag{3.25}$$

Therefore, the power contained in a single sideband in amplitude modulation is:

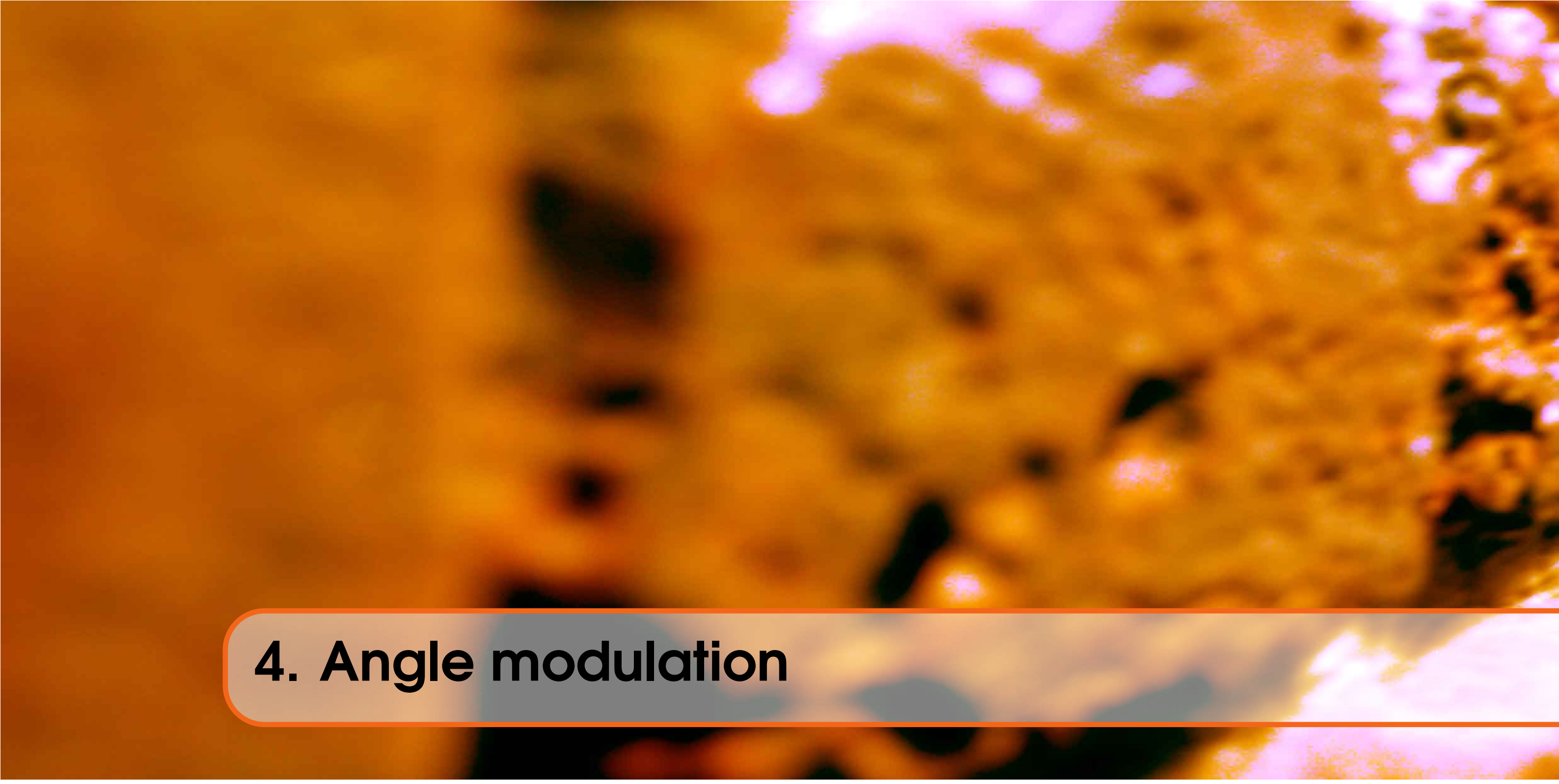
$$P_{\text{SSB}} = \frac{m^2 P_c}{4} \tag{3.26}$$

Interpretation

In amplitude modulation, the carrier consumes a large portion of the total transmitted power since it does not contain any information. Only the sidebands carry the message signal. Thus, in standard AM, the transmission efficiency is low, and techniques such as **single sideband (SSB)** or **double sideband suppressed carrier (DSB-SC)** are used to improve power efficiency by reducing or eliminating the carrier component.

3.4 Conclusion

In this chapter, we examined the essential principles of Amplitude Modulation (AM) and its various forms used in analog communication systems. Beginning with the basic concept of modulation, we learned how a message signal can be transferred to a higher frequency range suitable for transmission. We then analyzed the structure and operation of different AM types: DSB-SC (Double Sideband Suppressed Carrier), which improves power efficiency by eliminating the carrier, Conventional AM, which simplifies demodulation at the cost of higher power consumption, and SSB (Single Sideband), which optimizes bandwidth usage and reduces interference. The study of demodulation techniques further illustrated how information can be accurately recovered from modulated signals using synchronous and envelope detection methods. By understanding amplitude modulation and its practical variants, students are now equipped with the knowledge to analyze and design analog transmission systems. This chapter serves as a bridge between basic RF signal concepts and more advanced topics such as angle modulation and digital communication techniques presented in subsequent sections.



4. Angle modulation

4.1 Introduction

While amplitude modulation varies the amplitude of a carrier wave to convey information, angle modulation alters the phase or frequency of the carrier instead. This technique represents a major evolution in analog communication, offering superior performance in terms of noise immunity, signal quality, and transmission efficiency. This chapter introduces the fundamental principles of angle modulation, which includes Frequency Modulation (FM) and Phase Modulation (PM). Both methods are based on varying the instantaneous angle of the carrier wave in proportion to the message signal but differ in the specific parameter being controlled-frequency in FM and phase in PM. The first section discusses the mathematical representation of FM and PM signals, highlighting the relationship between the modulating signal and the instantaneous frequency or phase deviation. The analysis includes waveform behavior and the dependence of modulation index on the signal characteristics. The next section explores the spectral characteristics of angle-modulated signals, examining how bandwidth and sidebands are affected by the modulation index. The Carson's rule and Bessel functions are introduced to estimate the spectral components and occupied bandwidth of FM and PM signals. Finally, the chapter presents the implementation of angle modulators and demodulators, covering both analog and practical circuit realizations. Students will learn how FM and PM signals are generated using reactance modulators, varactor diodes, and phase modulators, and how they are demodulated using frequency discriminators and phase-locked loops (PLL). By the end of this chapter, students will have a complete understanding of how angle modulation techniques enhance signal robustness and are widely used in high-quality radio, television, and satellite communication systems.

4.2 Representation of FM and PM signals

An angle modulated signal in general can be written as,

$$u(t) = A_c \cos(\theta(t))$$

$\theta(t)$ is the phase of the signal, and its instantaneous frequency $f_i(t)$ is given by,

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t) \tag{4.1}$$

Since $u(t)$ is a bandpass signal, it can be represented as,

$$u(t) = A_c \cos(2\pi f_c t + \phi(t)) \tag{4.2}$$

and, therefore,

$$f_i(t) = f_c + \frac{1}{2\pi}\frac{d}{dt}\phi(t) \tag{4.3}$$

If $m(t)$ is the message signal then,

- in a PM system we have,

$$\phi(t) = k_p m(t) \tag{4.4}$$

- in FM system we have,

$$f_i(t) - f_c = k_f m(t) = \frac{1}{2\pi}\frac{d}{dt}\phi(t) \tag{4.5}$$

where k_p and k_f are phase and frequency deviation sonstants. From the above relationships we have,

$$\phi(t) = \begin{cases} k_p m(t), & PM \\ 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau, & FM \end{cases} \tag{4.6}$$

The first interesting result observed from equation 3.5 is that if we phase modulate the carrier with the integral of message, it is equivalent to frequency modulation of the carrier with the original message. On the other hand, equation 4.6 can be expressed as,

$$\frac{d}{dt}\phi(t) = \begin{cases} k_p \frac{d}{dt} m(t), & PM \\ 2\pi k_f m(t), & FM \end{cases} \tag{4.7}$$

which shows that if we frequency modulate the carrier with the derivative of a message, the result is equivalent to phase modulation of the carrier with the message itself.
The demodulation of an FM signal involves finding the instanteneous frequency of the modulated signal and then subtracting the carrier ffrequancy from it. In the demodulation of PM, the demodulation process is done by finding the phase of the signal and then recovering $m(t)$. The maximum phase deviation in a PM system is given by,

$$\Delta\phi_{max} = k_p max[|m(t)|] \tag{4.8}$$

and the maximum frequency deviation in an FM system is given by,

$$\Delta f_{max} = k_f max[|m(t)|] \tag{4.9}$$

Exercise 4.1 The message signal,

$$m(t) = a\cos(2\pi f_m t)$$

is used to either frequency modulate or phase modulate the carrier $A_c\cos(2\pi f_c t)$. Find the modulated signal in each case. ■

Solution
In PM we have,

$$\phi(t) = k_p m(t) = k_p a\cos(2\pi f_m t) \tag{4.10}$$

In FM we have,

$$\phi(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau = \frac{k_f a}{f_m}\sin(2\pi f_m t) \tag{4.11}$$

Therefore, the modulated signals will be,

$$u(t) = \begin{cases} A_c\cos(2\pi f_c t + k_p a\cos(2\pi f_m t)), & PM \\ A_c\cos(2\pi f_c t + \frac{k_f a}{f_m}\sin(2\pi f_m t)), & FM \end{cases} \tag{4.12}$$

By defining,

$$\beta_p = k_p a \tag{4.13}$$

$$\beta_f = \frac{k_f a}{f_m} \tag{4.14}$$

we have,

$$u(t) = \begin{cases} A_c\cos(2\pi f_c t + \beta_p\cos(2\pi f_m t)), & PM \\ A_c\cos(2\pi f_c t + \beta_f\sin(2\pi f_m t)), & FM \end{cases} \tag{4.15}$$

The parameters β_p and β_f are called the modulation indices of the PM and FM systems respectively. We can extend the definition of the modulation index for a general nonsinusoidal signal $m(t)$ as,

$$\beta_p = k_p max[|m(t)|] \tag{4.16}$$

$$\beta_f = \frac{k_f max[|m(t)|]}{W} \tag{4.17}$$

where W denotes the bandwidth of the message signal $m(t)$. In terms of the maximum phase and frequency deviation $\Delta\phi_{max}$ and Δf_{max}, we have,

$$\beta_p = k_p\Delta\phi_{max} \tag{4.18}$$

$$\beta_f = \frac{\Delta f_{max}}{W} \tag{4.19}$$

4.2.1 Spectral characteristics of angle modulated signals and Bessel functions

We will study the spectral characteristics of an angle modulated signal in three cases: when the modulating signal is a sinusoidal signal, when the modulating signal is a general periodic signal, and when the modulating signal is a general nonperiodic signal.

Angle modulation by a sinusoidal signal

Let us begin with the case where the message signal is a sinusoidal signal. As we have seen, in this case for both FM and PM, we have.

$$u(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t) \tag{4.20}$$

where β is the modulation index that can be either β_p or β_f. Therefore, the modulated signal can be written as,

$$u(t) = \Re(A_c e^{j2\pi f_c t} e^{j\beta \sin 2\pi f_m t}) \tag{4.21}$$

Since $\sin 2\pi f_m t$ is periodic width period $T_m = \frac{1}{f_m}$, the same is true for the complex exponential signal.

$$e^{j\beta \sin 2\pi f_m t}$$

Therefore, it can be expanded in Fourier series representation. The Fourier series coefficients are obtained from the integral.

$$\begin{aligned} c_n &= f_m \int_0^{\frac{1}{f_m}} e^{j\beta \sin 2\pi f_m t} e^{j2\pi n f_m t} dt \\ &= \frac{1}{2\pi} \int_0^{2\pi} e^{j(\beta \sin u - nu)} du \end{aligned} \tag{4.22}$$

This latter integral is a well-Known integral known as Bessel function of the first kind of order n and is denoted by $J_n(\beta)$. Therefore, we have the Fourier series for the complex exponential as,

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t} \tag{4.23}$$

By substituting equation 4.23 in equation 3.20, we obtain,

$$\begin{aligned} u(t) &= \Re\left(A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t} e^{j2\pi f_c t}\right) \\ &= \sum_{n=-\infty}^{\infty} A_c J_n(\beta) \cos(2\pi(f_c + n f_m)t) \end{aligned} \tag{4.24}$$

Equation 4.24 shows that even in this very simple case, where the modulating signal is a sinusoid of frequency f_m, the amgle modulated signal contains all frequencies of the form $f_c + n f_m$ for $n = 0, \pm 1, \pm 2, \ldots$ Therefore, the actual bandwidth of the modulated signal is infinite. However, the amplitude of the sinusoidal components of frequencies $f_c \pm n f_m$ for large n is very small. Hence, we can define a finite effective bandwidth for the modulated signal. A series expansion for the Bessel function is given by,

$$J_n(\beta) = \sum_{n=0}^{\infty} \frac{(-1)^k \left(\frac{\beta}{2}\right)^{n+2k}}{k!(k+n)!} \tag{4.25}$$

The above expansion shows that for small β, we can use the approximation,

$$J_n(\beta) \approx \frac{\beta^n}{2^n n!} \tag{4.26}$$

Thus for a small modulation index β, only the first sideband corresponding to $n = 1$ is of importance. Also, using the above expansion, it is easy to verify the following symmetry properties of the Bessel function.

$$J_{-n}(\beta) = \begin{cases} J_n(\beta), & n \ even \\ -J_n(\beta), & n \ odd \end{cases} \tag{4.27}$$

Plots for $J_n(\beta)$ for various values of n are given in Figure 4.1.

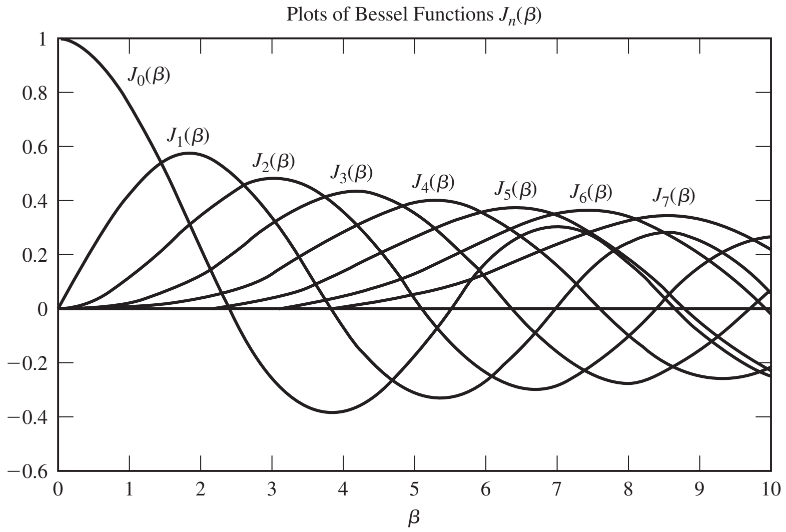


Figure 4.1: Bessel function for various values of n

and a table of the values of the Bessel function is given in Table 4.2.

n	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.5$	$\beta = 1$	$\beta = 2$	$\beta = 5$	$\beta = 8$	$\beta = 10$
0	0.997	0.990	0.938	0.765	0.224	−0.178	0.172	−0.246
1	0.050	0.100	0.242	0.440	0.577	−0.328	0.235	0.043
2	0.001	0.005	0.031	0.115	0.353	0.047	−0.113	0.255
3				0.020	0.129	0.365	−0.291	0.058
4				0.002	0.034	0.391	−0.105	−0.220
5					0.007	0.261	0.186	−0.234
6					0.001	0.131	0.338	−0.014
7						0.053	0.321	0.217
8						0.018	0.223	0.318
9						0.006	0.126	0.292
10						0.001	0.061	0.207
11							0.026	0.123
12							0.010	0.063
13							0.003	0.029
14							0.001	0.012
15								0.004
16								0.001

Figure 4.2: Bessel function values

Exercise 4.2 Let the carrier be given by $c(t) = 10\cos(2\pi f_c t)$. Further assume that the message is used to frequency modulate the carriere with $k_f = 50$. Find the expression for the modulated signal and determine how many harmonics should be selected to contain 99% of the modulated signal power. ■

Solution

$$P_c = \frac{{A_c}^2}{2} = \frac{100}{2} = 50 \tag{4.28}$$

The modulated signal is represented by,

$$\begin{aligned} u(t) &= 10\cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} \cos(20\pi\tau)d\tau\right) \\ &= 10\cos\left(2\pi f_c t + \frac{50}{10}\sin(20\pi t)\right) \\ &= 10\cos\left(2\pi f_c t + 5\sin(20\pi t)\right) \end{aligned} \tag{4.29}$$

The modulation index is given by,

$$\beta = k_f \frac{max[|m(t)|]}{f_m} = 5 \tag{4.30}$$

and therfore, the FM modulated signal is,

$$\begin{aligned} u(t) &= \sum_{n=-\infty}^{\infty} A_c J_n(\beta)\cos(2\pi(f_c + n f_m)t) \\ &= \sum_{n=-\infty}^{\infty} 10 J_n(5)\cos(2\pi(f_c + 10n)t) \end{aligned} \tag{4.31}$$

It is seen that the frequency content of the modulated signal is concentrated at frequencies of the form $f_c + 10n$ for various n. To make sure that at least 99% of the total power is within the effective bandwidth, we have to choose k large enough such that,

$$\sum_{n=-k}^{k} \frac{100{J_n}^2(5)}{2} \geq 0.99 * 50 \tag{4.32}$$

This is a nonlinear equation and its solution can be found by trial and error and by using tables of the Bessel functions. Of course, in finding the solution to this equation we have to employ the symmetry properties of Bessel function given in equation 4.27.
Using the propreties we have

$$50\left[{J_0}^2(5) + 2\sum_{n=1}^{k} {J_n}^2(5)\right] \geq 49.5 \tag{4.33}$$

Starting with small values of k and increasing it, we see that the smallest value of $k = 6$. Therfore, taking frequencies $f_c \pm 10k$ for $0 \leq k \leq 6$ guarantees that 99% of the power of the modulated signal has been included and only one per cent has been left out. This means that, if the modulated signal is passed through an ideal bandpass filter centred at f_c with a band width of at least $120Hz$, only 1% of the signal power will be eliminated. This gives us a practical way to define the effective bandwidth of the angle modulated signal as being $120Hz$. Figure 4.3 shows the frequencies present in the effective bandwidth of the modulated signal. In general the effective bandwidth of an angle

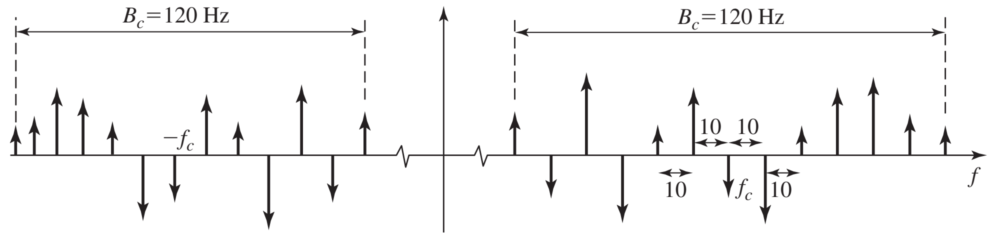


Figure 4.3: The harmonics present inside the effective bandwidth

modulated signal, which contains at least 98% of the signal power, is given by the relation.

$$\beta_c = 2(\beta + 1) f_m \tag{4.34}$$

where β is the modulation index and f_m is the frequency of the sinusoidal message signal. It is instructive to study the effect of the amplitude and frequency of the sinusoidal message signal on the bandwidth and the number of harmonics in the modulated signal. Let the message signal be given by,

$$m(t) = a\cos(2\pi f_m t) \tag{4.35}$$

The bandwidth of the modulated signal is given by,

$$B_c = 2(\beta + 1) f_m = \begin{cases} 2(k_p a + 1) f_m, & PM \\ 2\left(\dfrac{k_f a}{f_m} + 1\right) f_m, & FM \end{cases} \tag{4.36}$$

or,

$$B_c = \begin{cases} 2(k_p a + 1) f_m, & PM \\ 2\left(k_f a + f_m\right), & FM \end{cases} \tag{4.37}$$

Equation 4.44 shows that increasing a, the amplitude of the modulating signal, in PM and AM has almost the same effect on increasing the bandwidth B_c. On the other hand, increasing f_m, the frequency of the message signal, has a more profound effect in increasing the bandwidth of a PM signal as compared to an FM signal. In both PM and FM the bandwidth B_c increases by increasing f_m, but in PM this increase is a proportional increase and in FM this is only an additive increase, which in most cases of interest, (for large β) is not substantial.

Angle modulation by a periodic message signal

To generalize the preceding results, we now consider angle modulation by an arbitrary periodic message signal $m(t)$. Let us consider a PM modulated signal where,

$$u(t) = A_c \cos(2\pi f_c t + \beta m(t)) \tag{4.38}$$

We can write this as,

$$u(t) = A_c * \Re[e^{j2\pi f_c t} e^{j\beta m(t)}] \tag{4.39}$$

We are assuming that $m(t)$ is periodic with period $T_m = \dfrac{1}{f_m}$. Therefore, $e^{j\beta m(t)}$ will be a periodic signal with the same period, and we can find its Fourier series expansion as,

$$e^{j\beta m(t)} = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_m t} \tag{4.40}$$

where,

$$\begin{aligned} c_n &= \frac{1}{T_m}\int_0^{T_m} e^{j\beta m(t)}e^{j2\pi n f_m t}dt \\ &= \frac{1}{2\pi}\int_0^{2\pi} e^{j[\beta m(\frac{n}{2\pi f_m})-nu]}du|_{u=2\pi f_m t} \end{aligned} \tag{4.41}$$

and,

$$\begin{aligned} u(t) &= A_c * \Re \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_m t}e^{j2\pi f_c t} \\ u(t) &= A_c \sum_{n=-\infty}^{\infty} |c_n|\cos(2\pi(f_c+nf_m)t+\angle c_n) \end{aligned} \tag{4.42}$$

It is seen again that the modulated signal contains all frequencies of the form $f_c + nf_m$. The detailed treatment of the spectral characteristics of an angle modulated signal for a general nonperiodic deterministic message signal $m(t)$ is quite involved due to the nonlinear nature of the modulation process. However, there exists an approximate relation for the effective bandwidth of the modulated signal, known as the Carson's rule, and given by,

$$B_c = 2(\beta+1)W \tag{4.43}$$

where β is the modulation index defined as,

$$B_c = \begin{cases} k_p max[|m(t)|], & PM \\ \dfrac{k_f max[|m(t)|]}{W}, & FM \end{cases} \tag{4.44}$$

and W is the bandwidth of the message signal $m(t)$. Since in wideband FM the value of β is usuallly around 5 or more, it is seen that the bandwidth of angle modulated signal is much greater than the bandwidth of various amplitude modulation schemes, which is either W (in SSB) or 2W (in DSB or conventional AM).

4.2.2 Implimentation of angle modulatiors and demodulators

Any modulation and demodulation process involves the generation of new frequencies that were not present in the input signal. This is true for both amplitude and angle modulation systems. This means that, if we interpret the modulator as a system with the message signal $m(t)$ as the input and with the modulated signal $u(t)$ as the output, this system has frequencies in its output that were not present in the input. Therefore, a modulator (and demodulator) can not be modeled as a linear time invariant system because a linear time invariant system can not produce any frequency components in the output that are not present in the input signal. Angle modulations are, in general, time invaring and nolinear systems. One method for generating an FM signal directly is to design an oscillator whose frequency changes with the input voltage. When the input voltage is zero, the oscillator generates a sinusoid with frequency f_c, and when the input voltage changes, this frequency changes accordingly. There are to approaches to designing such an oscillator, usually called VCO or voltage controlled oscillator. One approach is to use a varactor diode. A varactor diode is a capacitance changes with the applied voltage. Therefore, if this capacitor is used in the tuned circuit of the oscillator and the message signal is applied to it, the frequency of the tuned circuit, and the oscillator, will change in accordance with the message signal. Let us assume that the inductance of the inductor in the tuned circuit of Figure 4.4 is L_0 and the capacitance of the varactor diode is given as,

$$C(t) = C_0 + k_0 m(t) \tag{4.45}$$

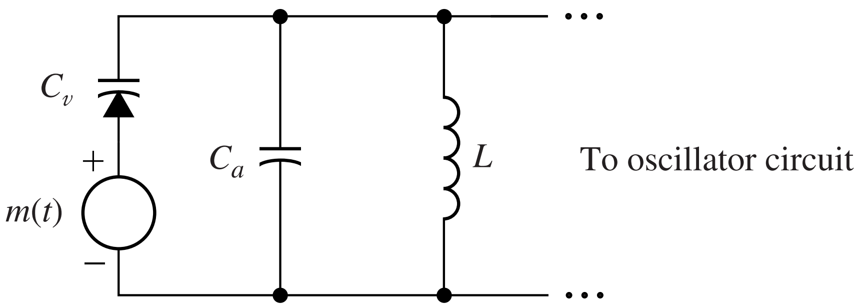


Figure 4.4: Varactor diode implementation in an angle modulator]

when $m(t) = 0$, the frequency of the tuned circuit is given by $f_c = \frac{1}{2\pi\sqrt{L_0 C_0}}$. In general, for nonzero $m(t)$ we have,

$$
\begin{aligned}
f_i(t) &= \frac{1}{2\pi\sqrt{L_0(C_0 + k_0 m(t))}} \\
f_i(t) &= \frac{1}{2\pi\sqrt{L_0 C_0}} \frac{1}{\sqrt{1 + \frac{k_0}{C_0} m(t)}} \\
f_i(t) &= f_c \frac{1}{\sqrt{1 + \frac{k_0}{C_0} m(t)}}
\end{aligned}
\tag{4.46}
$$

Assuming that,

$$\varepsilon \simeq \frac{k_0}{C_0} m(t) \ll 1 \tag{4.47}$$

and using the approximations,

$$\sqrt{1+\varepsilon} \approx 1 + \frac{\varepsilon}{2} \tag{4.48}$$

$$\frac{1}{1+\varepsilon} \approx 1 - \varepsilon \tag{4.49}$$

$$f_i(t) = f_c \left(1 - \frac{k_0}{2C_0} m(t)\right) \tag{4.50}$$

which is the relation for a frequency modulated signal.
Another approach for generating an angle modulated signal is to first generate a narrowband angle modulated signal, and then change it to a wideband signal. This method is usually known as the **indirect method** for generation of FM and PM signals. Due to the similarity of conventional AM signals, generation of narrowband angle modulated signals is straightforward. In fact any modulator for conventional AM generation can be easily modified to generate a narrowband angle modulated signal. Figure 4.5 shows the block diagram of a narrowband angle modulator. The next step is to use the narrowband angle modulated signal to generate a wideband angle modulated signal.

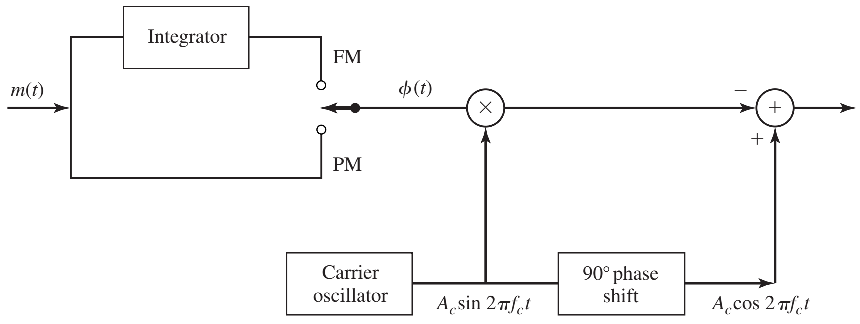


Figure 4.5: Generation of narrowband angle modulated signal

Figure 4.6 shows the block diagram of a system that generates wideband angle modulated signals from narrowband angle modulated signals. The first stage of such a system is, of course, a

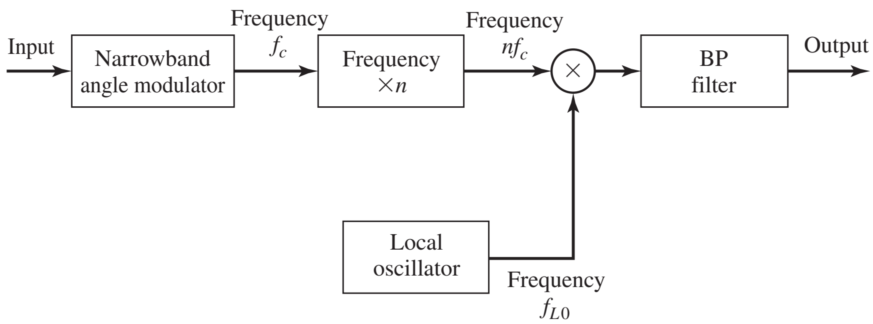


Figure 4.6: Indirect generation of angle modulated signals

narrowband angle modulator such as the one shown in Figure 4.5. The narrowband angle modulated signal enters a frequency multiplier that multiplies the instantaneous frequency of the input by some constant n. This is usually through a bandpass filter tuned to the desired central frequency. If the narrowband modulated signal is represented by,

$$u_n(t) = A_c \cos(2\pi f_c t + \phi(t)) \tag{4.51}$$

the output of the frequency multiplier(output of the band pass filter) is given by,

$$y(t) = A_c \cos(2\pi n f_c t + n\phi(t)) \tag{4.52}$$

In general, this is, of course, a wideband angle modulated signal. However, there is no guarantee that the carrier frequency of this signal, nf_c, will be the desired carrier frequency. The last stage of the modulator performs an up or down conversion to shift the modulated signal to the desired center frequency. This stage consists of a mixer and a bandpass filter. If the frequency of the local oscillator of the mixer is f_{LO} and we are using a down converter, the final wideband angle modulated signal is given by,

$$u(t) = A_c \cos(2\pi(nf_c - f_{LO})t + n\phi(t)) \tag{4.53}$$

Since we can freely choose n and f_{LO}, we can generate any modulation index at any desired carrier frequency by this method. FM demodulators are implemented by generating an AM signal whose amplitude is proportional to the instantaneous frequency of the FM signal, and then using an AM demodulator to recover the message signal. To implement the first step; i.e, transforming the FM signal to AM signal, it ise enough to pass the FM signal through an LTI system whose frequency response is approximately a straight line in the frequency band of the FM signal. If the frequency response of such a system is given by,

$$|H(f)| = V_0 + k(f - f_c), \qquad |f - f_c| < \frac{B_c}{2} \tag{4.54}$$

and if the input to the system is,

$$u(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right) \tag{4.55}$$

then, the output will be the signalm

$$u(t) = A_c(V_0 + kk_f m(t)) \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right) \tag{4.56}$$

The next step is to demodulate this signal to obtain $A_c(V_0 + kk_f m(t))$, from which the message $m(t)$ can be recovered. Figure 4.7 shows a block diagram of these two steps. There exist many circuits

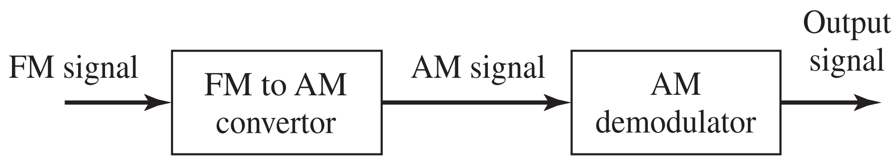


Figure 4.7: A general FM demodulator

that can be used to implement the first stage of an FM demodulator; i.e., FM to AM conversion. One such condidate is a simple differentiator with,

$$|H(f)| = 2\pi f \tag{4.57}$$

Another candidate is the rising half of the frequency characteristics of a tuned circuit as shown in Figure 4.8.

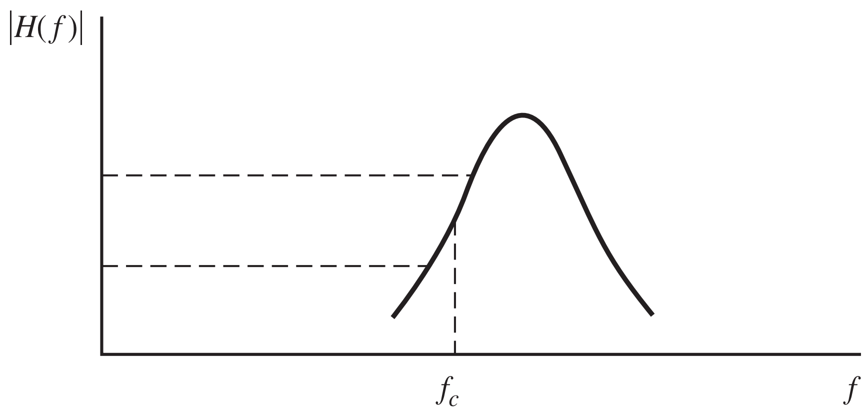


Figure 4.8: A tuned circuit used in an FM demodulator

Such a circuit can be easily implemented, but usually the linear region of the frequency characteristic may not be wide enough. To obtain a linear characteristic over a wider range of frequencies, usually two circuits tuned at two frequencies, f_1 and f_2, are connected in a configuration which is known as a balanced discriminator. Abalanced discriminator with the corresponding frequency characteristics is shown in Figure 4.9. The FM demodulation methods described here that transform the FM signal into an AM signal have a bandwidth equal to the channel bandwidth B_c occupied by the FM signal. Consequently, the noise that is passed by the demodulator is the noise contained within B_c.

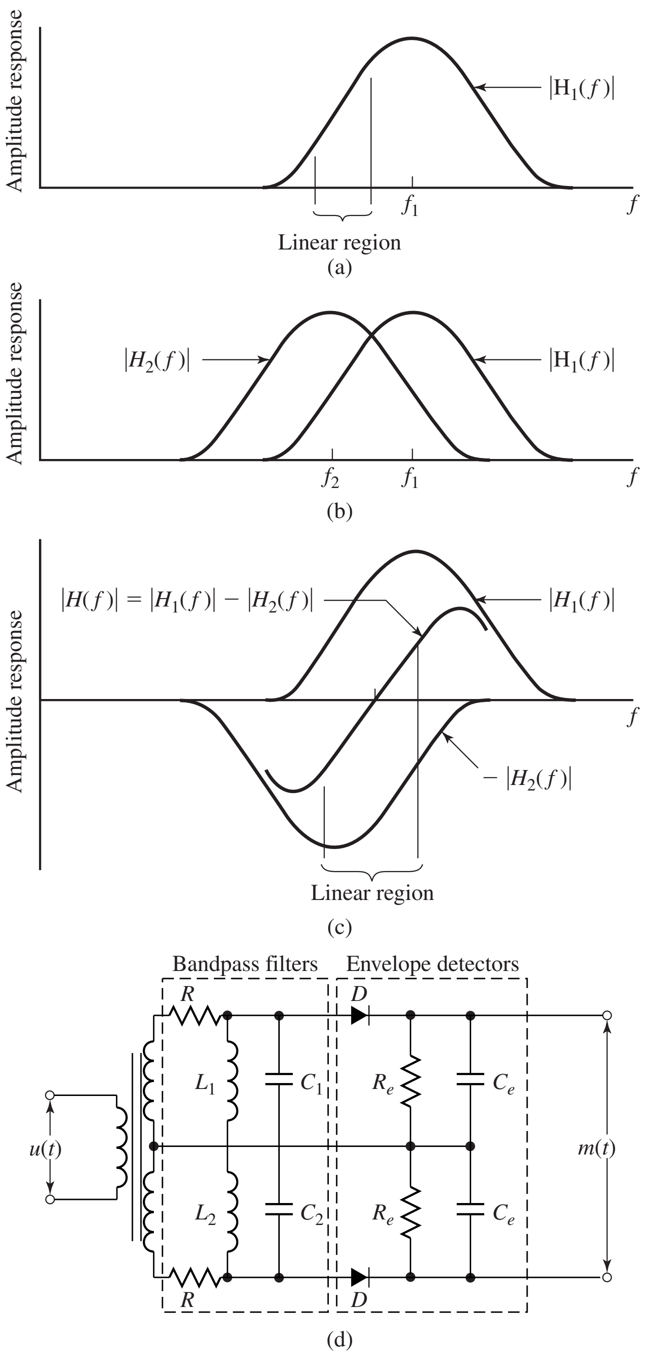


Figure 4.9: A balanced discriminator and corresponding frequency response

A totally different approach to FM signal demodulation is to use feedback in the FM demodulator to narrow the bandwidth of the FM detector and to reduce the noise power at the output of the demodulator. Figure 4.10 illustraates a system in which the FM discrimination is placed in the feedback branch of a feedback system that employs a voltage controlled oscillator (VCO) path. The bandwidth of the discriminator and the subsequent lowpass filter is designed to match

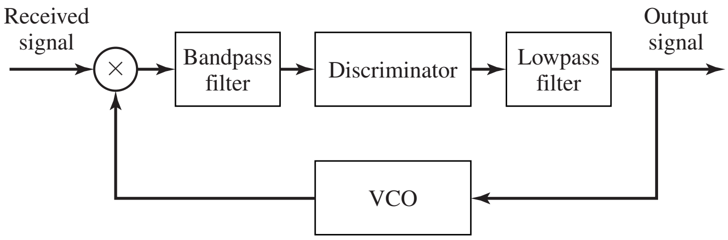


Figure 4.10: Block diagram of FMFB demodulator

the bandwidth of the message signal $m(t)$. The output of the lowpass filter is the desired message signal. This type of FM demodulator is called an FM *demodulator with feedback* (FMFB). An alternative to FMFB demodulator is the use of the phase locked loop (PLL), as shown in Figure 4.11. The input to the PLL is the angle modulated signal (we neglect the presence of noise in this discussion)

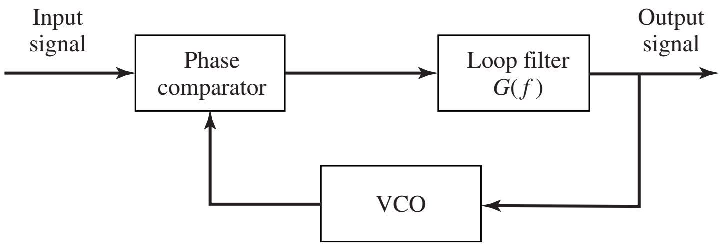


Figure 4.11: Block diagram of PLL-FM demodulator

$$u(t) = A_c \cos(2\pi f_c t + \pi) \tag{4.58}$$

where, for FM,

$$\phi(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau) d\tau \tag{4.59}$$

The VCO generates a sinusoid of a fixed frequency, in this case the carrier frequency f_c, in the absence of an input control voltage. Now, suppose that the control voltage to the VCO is the output of the loop filter, denoted as $v(t)$. Then, the instantaneous frequency of the VCO is,

$$f_v(t) = f_c + k_v v(t) \tag{4.60}$$

where k_v is a deviation constant with units of $Hz/volt$. Consequently, the VCO output may be expressed as,

$$y_v(t) = A_v \sin(2\pi f_c t + \phi_v(t)) \tag{4.61}$$

where,

$$\phi_v(t) = 2\pi k_v \int_{-\infty}^{t} v(\tau) d\tau \tag{4.62}$$

The phase comparator is basically a multipier and filter that rejects the signal component centered at $2f_c$. Hence, its output may be expressed as,

$$e(t) = \frac{1}{2}A_v A_c \sin(\phi(t) - \phi_v(t)) \tag{4.63}$$

where the difference, $\phi(t) - \phi_v(t) \equiv \phi_e(t)$, constitutes the phase error. The signal $e(t)$ is the input to the loop filter. Let us assume that the PLL is in lock, so that the phase error is small. Then,

$$\sin[\phi(t) - \phi_v(t)] \approx \phi(t) - \phi_v(t) = \phi_e(t) \tag{4.64}$$

Under this condition, we may deal with the linearized model of the PLL, shown in Figure 4.12.

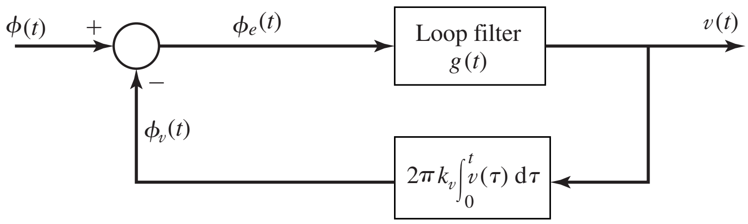


Figure 4.12: Linearized PLL

We may express the phase error as,

$$\phi_e(t) = \phi(t) - 2\pi k_v \int_0^t v(\tau)d\tau \tag{4.65}$$

or, equivalently, either as,

$$\frac{d}{dt}\phi_e(t) + 2\pi k_v v(t) = \frac{d}{dt}\phi(t) \tag{4.66}$$

$$\frac{d}{dt}\phi_e(t) + 2\pi k_v \int_0^\infty \phi_e(\tau)g(t-\tau)d\tau = \frac{d}{dt}\phi(t) \tag{4.67}$$

The Fourier transform of the integro-differential equation in Equation 4.67 is,

$$(j2\pi f)\Phi_e(f) + 2\pi k_v \Phi_e(f)G(f) = (j2\pi f)\Phi(f) \tag{4.68}$$

and, hence,

$$\Phi_e(f) = \frac{1}{1 + \left(\frac{k_v}{jf}\right)G(f)}\Phi(f) \tag{4.69}$$

The corresponding equation for the control voltage to the VCO is,

$$V(f) = \Phi_e(f)G(f) = \frac{G(f)}{1 + \left(\frac{k_v}{jf}\right)G(f)}\Phi(f) \tag{4.70}$$

Now, suppose that we design $G(f)$ such as,

$$\left|\frac{k_v G(f)}{jf}\right| \gg 1 \tag{4.71}$$

in the frequency band $|f| < W$ of the message signal. Then from Equation 4.70 we have,

$$V(f) = \frac{j2\pi f}{2\pi k_v}\Phi(f) \tag{4.72}$$

or, equivalently,

$$v(t) = \frac{1}{2\pi k_v}\frac{d}{dt}\phi(t) = \frac{k_f}{k_v}m(t) \tag{4.73}$$

Since the control voltage of the VCO is proportional to the message signal, $v(t)$ is the demodulated signal. We observe that the output of the loop filter with frequency response $G(f)$ is the desired message signal. Hence, the bandwidth of $G(f)$ should be the same as the bandwidth W of the message signal. Consequently, the noise at the output of the loop filter is also limited to the bandwidth W. On the other hand, the output from the VCO is a wideband FM signal with an instantaneous frequency that follows the instantaneous frequency of the received FM signal. The major benefit of using feedback in FM signal demodulation is to reduce the thereshold effect that accurs when the input signal to noise ratio to the FM demodulator drops below a critical value.

4.3 Comparison between AM, FM, and PM

Amplitude Modulation (AM), Frequency Modulation (FM), and Phase Modulation (PM) are the three main types of analog modulation techniques used in communication systems. They differ in the parameter of the carrier wave that is varied according to the message signal:amplitude, frequency, or phase, respectively. The following table summarizes their key characteristics, including their mathematical expressions, modulation indices, and bandwidth dependencies.

Table 4.1: Comparison between AM, FM, and PM

Feature	**AM**	**FM**	**PM**
Definition	Amplitude of carrier varies.	Frequency of carrier varies.	Phase of carrier varies.
Modulated Parameter	A_c	f_c	θ
Equation	$s(t) = A_c[1 + m(t)]\cos(\omega_c t)$	$s(t) = A_c \cos(\omega_c t + \beta \sin \omega_m t)$	$s(t) = A_c \cos(\omega_c t + \beta \cos \omega_m t)$
Index (β)	$\mu = \frac{A_m}{A_c}$	$\beta = \frac{\Delta f}{f_m}$	$\beta = k_p A_m$
Dependence	On amplitude of $m(t)$.	On amplitude and frequency of $m(t)$.	On amplitude of $m(t)$.
Bandwidth	$2f_m$	$2(\Delta f + f_m)$	Similar to FM.

The table provides a clear comparison between the three main types of analog modulation: Amplitude Modulation (AM), Frequency Modulation (FM), and Phase Modulation (PM). In AM, the amplitude of the carrier signal varies according to the message signal, making it simple to implement but more sensitive to noise and amplitude distortion. FM and PM, on the other hand, vary the frequency and phase of the carrier respectively, providing better noise immunity and signal quality at the expense of a larger bandwidth and more complex circuitry. Overall, FM and PM are preferred in modern communication systems where signal quality is critical, while AM remains common in applications where simplicity and bandwidth efficiency are more important.

4.4 Conclusion

In this chapter, we explored the theory and applications of angle modulation, focusing on Frequency Modulation (FM) and Phase Modulation (PM) as the two main variants. Unlike amplitude modulation, these techniques encode information through variations in the carrier's frequency or phase, offering significant advantages in terms of noise resistance and transmission fidelity.

We analyzed the mathematical models of FM and PM signals and examined their spectral properties, noting how modulation index and deviation determine the occupied bandwidth. The study of Carson's rule provided a practical means to estimate the bandwidth requirements of angle-modulated systems.

Furthermore, we reviewed several implementation techniques for generating and detecting FM and PM signals, emphasizing the practical use of modulator circuits, discriminators, and PLL-based demodulators in real-world communication systems.

Through this study, students have gained an essential understanding of how angle modulation improves communication performance, paving the way for more advanced modulation schemes and modern digital techniques. Angle modulation thus represents a crucial step toward achieving efficient, high-fidelity, and noise-resilient communication in both analog and digital systems.

5. Effect of noise on analog communication syster

5.1 Introduction

In any analog communication system, noise is an unavoidable disturbance that degrades the quality of the transmitted signal. This chapter examines the influence of noise on different types of analog modulation. We first analyze its effect on linear modulation systems (AM, DSB, SSB), and then on angle modulation systems (FM and PM), to understand how the nature of the noise and the modulation technique affect the receiver performance and the signal to noise ratio (SNR).

5.1.1 Effect of noise on linear modulation systems

In this section we determine the signal to noise ratio (SNR) of the output of the receiver that demodulates the amplitude modulated signals. In evaluating the effect of noise on the various types of analog modulated signals, it is also interesting to compare the result with effect of noise on an equivalent baseband communication system. We begin the evaluation of the effect of noise on a baseband system.

Effect of Noise on a baseband system

Since baseband systems serve as a basis for comparison of various modulation systems, we begin with a noise analysis of a baseband system. In thise case there exists no demodulator, and the receiver consists only of a lowpass filter with bandwidth W. The noise power at the output of the receiver is,

$$P_{n_o} = \int_{-W}^{W} \frac{N_0}{2} df = N_0 W \tag{5.1}$$

If we denote the received power by P_R, the baseband SNR is given by,

$$\left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W} \tag{5.2}$$

Effect of Noise on DSB-SC AM

In DSB, we have,

$$u(t) = A_c \cos(2\pi f_c t + \phi_c) \tag{5.3}$$

and, therefore, the received signal at the output of the receiver noiselimiting filter is,

$$\begin{aligned} r(t) &= u(t) + n(t) \\ &= A_c m(t)\cos(2\pi f_c t + \phi_c) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t) \end{aligned} \tag{5.4}$$

Suppose we demodulate the received signal by first multiplying $r(t)$ by a locally generated sinusoid $\cos(2\pi f_c t + \phi)$, whre ϕ is the phase of the sinusoid, and then passing the product signal through an ideal lowpass filter having a bandwidth W. the multiplication of $r(t)$ with $\cos(2\pi f_c t + \phi)$ yields

$$\begin{aligned} r(t)\cos(2\pi f_c t + \phi) &= A_c m(t)\cos(2\pi f_c t + \phi_c)\cos(2\pi f_c t + \phi) + n(t)\cos(2\pi f_c t + \phi) \\ &= \frac{1}{2}A_c m(t)\cos(\phi_c - \phi) + \frac{1}{2}A_c m(t)\cos(4\pi f_c t + \phi + \phi_c) \\ &+ \frac{1}{2}[n_c(t)\cos\phi + n_s(t)\sin\phi] + \frac{1}{2}[n_c(t)\cos(4\pi f_c t + \phi) \\ &- n_s(t)\sin(4\pi f_c t + \phi)] \end{aligned} \tag{5.5}$$

The lowpass filter rejects the double frequency componemts and pass only the lowpass components. Hence, its output is,

$$y(t) = \frac{1}{2}A_c m(t)\cos(\phi_c - \phi) + \frac{1}{2}[n_c(t)\cos\phi + n_s(t)\sin\phi] \tag{5.6}$$

The effect of a phase difference between the transmitter and the receiver is a reduction equal to $\cos^2(\phi_c - \phi)$ in the received signal power. Thid can be avoided by employing a PLL. The effect of a PLL is to generate a sinusoid at the receiver with the same frequency and phase of the carrier. If a PLL is employed, then $\phi - \phi_c$, and the demodulator is called a coherent or synchronous demodulator. In our analysis in this section, we assume that we are employing a coherent demodulator. With this assumption and without loss of generality, we can assume $\phi = \phi_c = 0$, and Equation 5.6 reduces to

$$y(t) = \frac{1}{2}[A_c m(t) + n_c(t)] \tag{5.7}$$

Therefore, at the receiver output the message signal and the noise components are additive and we are able to define a meaningful SNR. The message signal power is given by,

$$P_0 = \frac{{A_c}^2}{4}P_m \tag{5.8}$$

and the noise power is given as,

$$P_{n_0} = \frac{1}{4}P_{n_c} = \frac{1}{4}P_n \tag{5.9}$$

where w have used the fact that the power contents of $n_c(t)$ and $n(t)$ are equal. The power spectral density of $n(t)$ is given by,

$$S_n(f) = \begin{cases} \frac{N_0}{2}, & |f - f_c| < W \\ 0, & otherwise \end{cases} \tag{5.10}$$

The noise power is,

$$P_n = \int_{-\infty}^{\infty} S_n(f)df = \frac{N_0}{2}4W = 2WN_0 \tag{5.11}$$

Now we can find the output SNR as,

$$\left(\frac{S}{N}\right)_0 = \frac{P_0}{P_{n_0}} = \frac{\frac{A_c{}^2}{4}P_m}{\frac{1}{4}2WN_0} = \frac{A_c{}^2 P_m}{2WN_0} \tag{5.12}$$

In this case, the received signal power is $P_R = \frac{A^2{}_c P_m}{2}$. Therefore, by using Equation 5.12, we obtain,

$$\left(\frac{S}{N}\right)_{0_{DSB}} = \frac{P_R}{WN_0} = \left(\frac{S}{N}\right)_b \tag{5.13}$$

It is seen that in DSB-SC AM, the output SNR is the same as the SNR for a baseband system. Therfore, DSB-SC AM does not provide any SNR improvement over a simple baseband communication system.

Effect of noise on SSB AM

In this case,

$$u(t) = A_c m(t)\cos(2\pi f_c t) \pm A_c \hat{m}(t)\sin(2\pi f_c t) \tag{5.14}$$

Therefore, the input to the demodulator is,

$$\begin{aligned} r(t) &= A_c m(t)\cos(2\pi f_c t) \pm A_c \hat{m}(t)\sin(2\pi f_c t) + n(t) \\ &= (A_c m(t) + n_c(t))\cos(2\pi f_0 t) + (\pm A_c \hat{m}(t) - n_s(t))\sin(2\pi f_c t) \end{aligned} \tag{5.15}$$

Here again we assume that demodulation occurs with an ideal phase reference. Hence, the output of the lowpass filter is the in-phase component(with a coefficient of $\frac{1}{2}$) of the obove signal,

$$y(t) = \frac{A_c}{2}m(t) + \frac{1}{2}n_c(t) \tag{5.16}$$

It is observed that, in this case, again the signal and the noise components are additive and a meaningful SNR at the receiver output can be defined. Parallel to our discussion of DSB we have,

$$P_0 = \frac{A_c{}^2}{4}P_m \tag{5.17}$$

$$P_{n_0} = \frac{1}{4}P_{n_c} = \frac{1}{4}P_n \tag{5.18}$$

The noise power spectral density in this case is $N_0/2$ over the bandwidth of the frontend filter at the receiver, which here has a bandwidth of W. For instance, in the USSB case, we have,

$$S_n(f) = \begin{cases} \frac{N_0}{2}, & f_c \leq |f| \leq f + f_c \\ 0, & otherwise \end{cases} \tag{5.19}$$

Therefore,

$$P_n = \int_{-\infty}^{\infty} S_n(f)df = \frac{N_0}{0}2W = WN_0 \tag{5.20}$$

Therefore,

$$\left(\frac{S}{N}\right)_{0_{SSB}} = \frac{P_0}{P_{n_0}} = \frac{{A_c}^2 P_m}{WN_0} \tag{5.21}$$

But , in This case,

$$P_R = P_U = {A_c}^2 P_m \tag{5.22}$$

and, therefore,

$$\left(\frac{S}{N}\right)_{0_{SSB}} = \frac{P_R}{WN_0} = \left(\frac{S}{N}\right)_b \tag{5.23}$$

Therefore, the SNR in a SSB system is equivalent to that of a DSB system.

Effect of noise on conventional AM

In conventional DSB AM, the modulated signal is,

$$u(t) = A_c[1 + am_n(t)]\cos(2\pi f_c t) \tag{5.24}$$

Therefore, the received signal at the input to the demodulator is,

$$r(t) = [A_c(1 + am_n(t)) + n_c(t)]\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t) \tag{5.25}$$

where a is the mdulation index and $m_n(t)$ is normalized so that its minimum value is -1. If a synchronous demodulator is employed, the situation is basically similar to the DSB case, except that we have $1 + am_n(t)$ instead of $m(t)$. Therefore, in this case, after mixing and lowpass filtring, we have,

$$y_1(t) = \frac{1}{2}[A_c(1 + am_n(t)) + n_c(t)] \tag{5.26}$$

However, in this case, the desired signal is $m(t)$, not $1 + am_n(t)$. The dc component in the demodulated waveform is removed by a dc blocking device and, hence, the lowpass filter output is,

$$y(t) = \frac{1}{2}A_c am_n(t) + \frac{n_c(t)}{2} \tag{5.27}$$

In this case, from equation 5.25, we obtain the received signal power P_R as,

$$P_R = \frac{{A_c}^2}{2}[1 + a^2 P_{m_n}] \tag{5.28}$$

where we have assumed that the message signal is zero mean. Now we can derive the output SNR for the coherent demodulator. From equation 5.27.

$$\begin{aligned}
\left(\frac{S}{N}\right)_{0_{AM}} &= \frac{\frac{{A_c}^2}{4} a^2 P_{m_n}}{\frac{1}{4} P_{n_c}} \\
&= \frac{{A_c}^2 a^2 P_{m_n}}{2WN_0} \\
&= \frac{a^2 P_{m_n}}{1 + a^2 P_{m_n}} \frac{\frac{{A_c}^2}{2}[1 + a^2 P_{m_n}]}{WN_0} \\
&= \frac{a^2 P_{m_n}}{1 + a^2 P_{m_n}} \frac{P_R}{WN_0} \\
&= \frac{a^2 P_{m_n}}{1 + a^2 P_{m_n}} \left(\frac{S}{N}\right)_b \\
&= \eta \left(\frac{S}{N}\right)_b
\end{aligned} \tag{5.29}$$

where we have used equation 5.2 and,

$$\eta = \frac{a^2 P_{m_n}}{1 + a^2 P_{m_n}} \tag{5.30}$$

denotes the modulation efficiency.

Exercise 5.1 The message signal $m(t)$ has a bandwidth of 10 KHz , a power of 16 W and maximum amplitude of 6. It is desirable to transmit this message to a destination via a channel with 80 dB attenuation and additive white noise with power spectral density $S_n(f) = \frac{N_0}{2} = 10^{-12}$ W/Hz, and achieve a SNR at the modulator output of at least 50 dB. What is the required transmitter power and channel bandwidth if the following modulation schemes are employed.

1 DSB AM
2 SSB AM
3 Conventional AM with modulation index equal to 0.8

■

Solution

We first determine $\left(\frac{S}{N}\right)_b$ as a basis of comparison.

$$\left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W} = \frac{P_R}{2 \times 10^{-12} \times 10^4} = \frac{10^8 P_R}{2}$$

Since the channel attenuation is 80 dB, the ratio of transmitted power P_T to received power P_R is,

$$10 \log \frac{P_T}{P_R} = 80$$

and, therfore,

$$P_R = 10^{-8} P_T$$

Hence,

$$\left(\frac{S}{N}\right)_b = \frac{10^8 \times 10^{-8} P_T}{2} = \frac{P_T}{2}$$

1- **For DSB AM**

$$\left(\frac{S}{N}\right)_0 = \left(\frac{S}{N}\right)_b = \frac{P_T}{2} \sim 50dB = 10^5$$

therefor,

$$\frac{P_T}{2} = 10^5 \Longrightarrow P_T = 2 \times 10^5 W \sim 200KW$$

and,

$$BW = 2W = 2 \times 10000 = 20000Hz \sim 20KHz$$

2- **For SSB AM**

$$\left(\frac{S}{N}\right)_0 = \left(\frac{S}{N}\right)_b = \frac{P_T}{2} = 10^5 \Longrightarrow P_T = 200KW$$

and,

$$BW = W = 10000Hz = 10KHz$$

1- **For conventional AM,** $a = 0.8$

$$\left(\frac{S}{N}\right)_0 = \eta \left(\frac{S}{N}\right)_b = \eta \frac{P_T}{2}$$

where η is the modulation efficiency given by,

$$\eta = \frac{a^2 P_{m_n}}{1 + a^2 P_{m_n}}$$

First we find P_{m_n}, the power content of the normalized message signal. Since $max|m(t)| = 6$, we have,

$$P_{m_n} = \frac{P_m}{(max|m(t)|)^2} = \frac{P_m}{36} = \frac{16}{36} = \frac{4}{9}$$

Hence,

$$\eta = \frac{0.8^2 \times \frac{4}{9}}{1 + 0.8^2 \times \frac{4}{9}} \approx 0.22$$

Therefore,

$$\left(\frac{S}{N}\right)_0 \approx 0.22 \frac{P_T}{2} = 0.11 P_T = 10^5$$

or,

$$P_T \approx 909 KW$$

The bandwidth of conventional AM is equal to the bandwidth of DSB AM; i.e, $BW = 2W = 20KHz$

5.1.2 Effect of noise on angle modulation

In this section, we will study the performance of angle modulated signals when contaminated by additive white Gaussian noise and compare this performance with the performance of amplitude modulated signals. The block diagram of the receiver for a general angle modulated signal is shown in Figure 5.1.

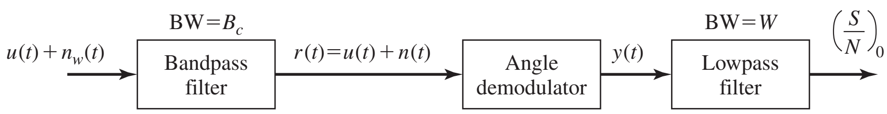


Figure 5.1: Block diagram of receiver for a general angle demodulated signal

The angle modulated signal is represented as,

$$u(t) = A_c \cos(2\pi f_c t + \phi(t)) = \begin{cases} A_c \cos(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau) d\tau), & FM \\ A_c \cos(2\pi f_c t + k_p m(t)), & PM \end{cases} \tag{5.31}$$

The additive white Gausian noise $n_w(t)$ is added to $u(t)$ and the result is passed through a noise limiting filter whose role is to remouve the out of band noise. The bandwidth of this filter is equal to the bandwidth of the modulated signal, and therefore, it passes the modulated signal without distortion. However, it eliminates the out of band noise and, hence, the noise output of the filter is a bandpass Gaussian noise denoted by $n(t)$. The output of this filter is,

$$\begin{aligned} r(t) &= u(t) + n(t) \\ &= u(t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t) \end{aligned} \tag{5.32}$$

As with conventional AN noise performance analysis, a precise analysis is quite involved due to the nonlinearity of the demodulation process. Let us make the assumption that the signal power is much higher than the noise power. Then, if the bandpass noise is represented as,

$$\begin{aligned} n(t) &= \sqrt{n_c{}^2(t) + n_s{}^2(t)} \cos\left(2\pi f_c t + \arctan\frac{n_s(t)}{n_c(t)}\right) \\ &= V_n(t)\cos(2\pi f_c t - \Phi_n(t)) \end{aligned} \tag{5.33}$$

where $V_n(t)$ and $\Phi_n(t)$ represent the envelope and phase of the bandpass noise process, respectively, the assumption that the signal is much larger than the noise means that,

$$P(V_n(t) \ll A_c) \approx 1 \tag{5.34}$$

Therefore, the phasor diagram of the signal and the noise are as shown in Figure 5.2.

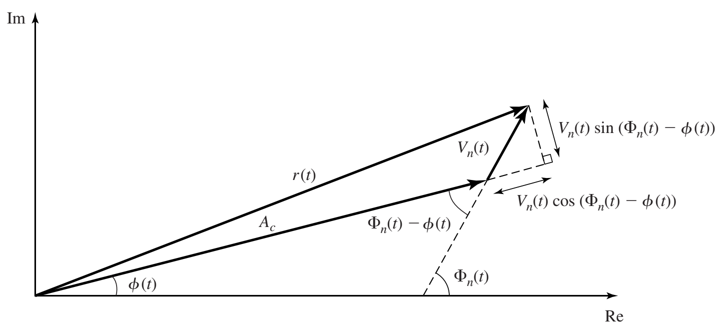


Figure 5.2: Phasor diagram of signal and noise in an angle modulated system

From this figure it is obvious that we can write,

$$\begin{aligned} r(t) &\approx [A_c + V_n(t)\cos(\Phi_n(t) - \phi(t))] \\ &\quad \times \cos\left(2\pi f_c t + \phi(t) + \arctan\frac{V_n(t)\sin(\Phi_n(t) - \phi(t))}{A_c + V_n(t)\cos(\Phi_n(t) - \phi(t))}\right) \\ &\approx [A_c + V_n(t)\cos(\Phi_n(t) - \phi(t))]\cos\left(2\pi f_c t + \phi(t) + \frac{V_n(t)}{A_c}\sin(\Phi_n(t) - \phi(t))\right) \end{aligned}$$

The demodulator processes this signal and, depending whether it is a phase or a frequency demodulator, its output will be the phase or the instantaneous frequency of this signal. Therefore, noting that,

$$\phi(t) = \begin{cases} k_p m(t), & PM \\ 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau, & FM \end{cases} \tag{5.35}$$

the output of the demodulator is given by,

$$\begin{aligned} y(t) &= \begin{cases} k_p m(t) + Y_n(t), & PM \\ k_f m(t) + \frac{1}{2\pi}\frac{d}{dt}Y_n(t), & FM \end{cases} \\ &= \begin{cases} k_p m(t) + \frac{V_n(t)}{A_c}\sin(\Phi_n(t) - \phi(t)), & PM \\ k_f m(t) + \frac{1}{2\pi}\frac{d}{dt}\frac{V_n(t)}{A_c}\sin(\Phi_n(t) - \phi(t)), & FM \end{cases} \end{aligned} \tag{5.36}$$

where, we have defined,

$$Y_n(t) = \frac{V_n(t)}{A_c}\sin(\Phi_n(t) - \phi(t)) \tag{5.37}$$

The first term in equations 5.5 - 5.7 is the desired signal component and the second term is the noise component. From this expression, we observe that the noise component is inversely proportional to the signal amplitude A_c. Hence, the higher the signal level, the lower will be the noise level. This is in agreement with the intuitive reasoning presented at the beginning of this section based on Figure 5.3. Note also that this is not the case with amplitude modulation. In AM system, the

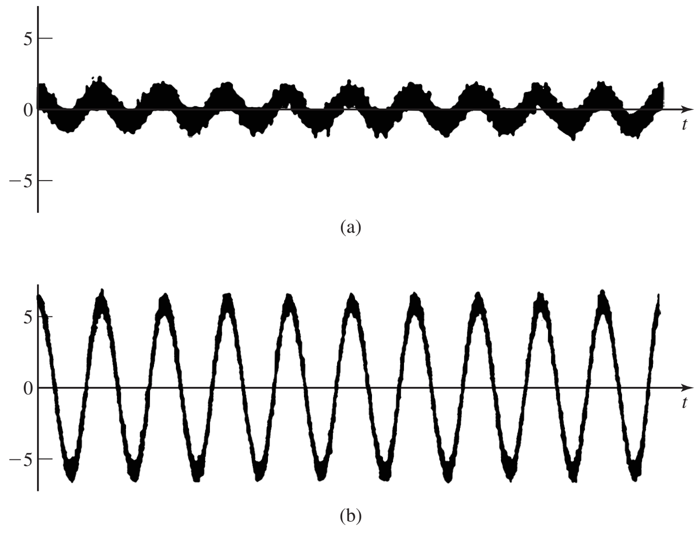


Figure 5.3: Effect of noise on the zero crossings of (a) low power and (b) high power modulated signals

noise component is independent of the signal component and a scaling of the signal power does not affect the received noise power.

Let us study the properties of the noise component given by,

$$
\begin{aligned}
Y_n(t) &= \frac{V_n(t)}{A_c}\sin(\Phi_n(t)-\phi(t)) \\
&= \frac{1}{A_c}[V_n(t)\sin\Phi_n(t)\cos\phi(t) - V_n(t)\cos\Phi_n(t)\sin\phi(t)] \\
&= \frac{1}{A_c}[n_s(t)\cos\phi(t) - n_c(t)\sin\phi(t)]
\end{aligned} \tag{5.38}
$$

The autocorrelation function of this process is given by,

$$
\begin{aligned}
E[Y_n(t+\tau)Y_n(t)] &= \frac{1}{{A_c}^2}E[R_{n_s}(\tau)\cos(\phi(t))\cos(\phi(t+\tau)) \\
+R_{n_c}(\tau)\sin(\phi(t+\tau))\sin(\phi(t))] &= \frac{1}{{A_c}^2}R_{n_c}(\tau)E[\cos(\phi(t+\tau)-\phi(t))]
\end{aligned} \tag{5.39}
$$

where we have used the fact that the noise process is stationary and $R_{n_c}(\tau) = R_{n_s}(\tau)$ and $R_{n_c n_s}(\tau) = 0$. Now we assume that the message $m(t)$ is a sample function of a zero-mean, stationary Gaussian process $M(t)$ with the autocorrelation function $R_M(\tau)$. Then, in both PM and FM modulation, $\phi(t)$ will also be a sample function of a zero-mean stationary, Gaussian process $\Phi(t)$. For PM this is obvious because,

$$
\Phi(t) = k_p M(t) \tag{5.40}
$$

and in the FM case, we have,

$$
\Phi(t) = 2\pi k_f \int_{-\infty}^{t} M(\tau)d\tau \tag{5.41}
$$

Noting that $\int_{-\infty}^{t}$ represents a linear time invariant operation it is seen that, in this case, $\Phi(t)$ is the output of an LTI system whose input is a zero-mean, stationary Gaussian process. Consequently $\Phi(t)$ will also be a zero-mean, stationary Gaussian process. At any fixed time t, the random variable $Z(t,\tau) = \Phi(t+\tau) - \Phi(t)$ is the difference between two jointly Gaussian random variables. Therefore, it is itself a Gaussian random variable with mean equal to zero and variance

$$
{\sigma_Z}^2 = E[\Phi^2(t+\tau)] + E[\Phi^2(t)] - 2R_\Phi(\tau) = 2[R_\Phi(0) - R_\Phi(\tau)] \tag{5.42}
$$

Now, using this result in equation 5.39 we obtain,

$$
\begin{aligned}
E[Y_n(t+\tau)Y_n(t)] &= \frac{1}{{A_c}^2}R_{n_c}(\tau)E[\cos(\Phi(t+\tau)-\Phi(t))] \\
&= \frac{1}{{A_c}^2}R_{n_c}(\tau)\Re[E\exp j(\Phi(t+\tau)-\Phi(t))] \\
&= \frac{1}{{A_c}^2}R_{n_c}(\tau)\Re[E\exp jZ(t,\tau)] \\
&= \frac{1}{{A_c}^2}R_{n_c}(\tau)\Re[\exp(-\frac{1}{2}{\sigma_z}^2)] \\
&= \frac{1}{{A_c}^2}R_{n_c}(\tau)\Re[\exp-(R_\Phi(0)-R_\Phi(\tau))] \\
&= \frac{1}{{A_c}^2}R_{n_c}(\tau)\exp-(R_\Phi(0)-R_\Phi(\tau))
\end{aligned} \tag{5.43}
$$

This result shows that under the assumption of a stationary Gaussian message, the noise process at the output of the demodulator is also a stationary process whose autocorrelation function is given above and whose power spectral density is,

$$\begin{aligned} S_Y(f) &= \mathscr{F}[R_Y(\tau)] \\ &= \mathscr{F}[\frac{1}{{A_c}^2}R_{n_c}(\tau)\exp-(R_\Phi(0)-R_\Phi(\tau))] \\ &= \frac{\exp-R_\Phi(0)}{{A_c}^2}\mathscr{F}[R_{n_c}(\tau)\exp(R_\Phi(\tau))] \\ &= \frac{\exp-R_\Phi(0)}{{A_c}^2}\mathscr{F}[R_{n_c}(\tau)g(\tau)] \\ &= \frac{\exp-R_\Phi(0)}{{A_c}^2}S_{n_c}(f)\star G(f) \end{aligned} \tag{5.44}$$

where $g(\tau)=\exp R_\Phi(\tau)$ and $G(f)$ is its Fourier transform. It can be shown that bandwidth of $g(\tau)$ is half the bandwidth B_c of the angle modulated signal, which for high modulation indices is much larger than W, the message bandwidth. Since the bandwidth of the angles modulated signal is defined as the frequencies that contain $98\%-99\%$ of the signal power, $G(t)$ is very small in the neighborhood of $|f|=\dfrac{B_c}{2}$ and, of course,

$$S_{n_c}(f)=\begin{cases} N_0, & |f|<\frac{B_c}{2} \\ 0, & otherwise \end{cases} \tag{5.45}$$

A typical example of $G(f), S_{n_c}(f)$ and the result of their convolution is shown in Figure 5.4. Because $G(f)$ is very small in the neighborhood of $|f|=\frac{B_c}{2}$, the resulting $S_Y(f)$ has almost a flat

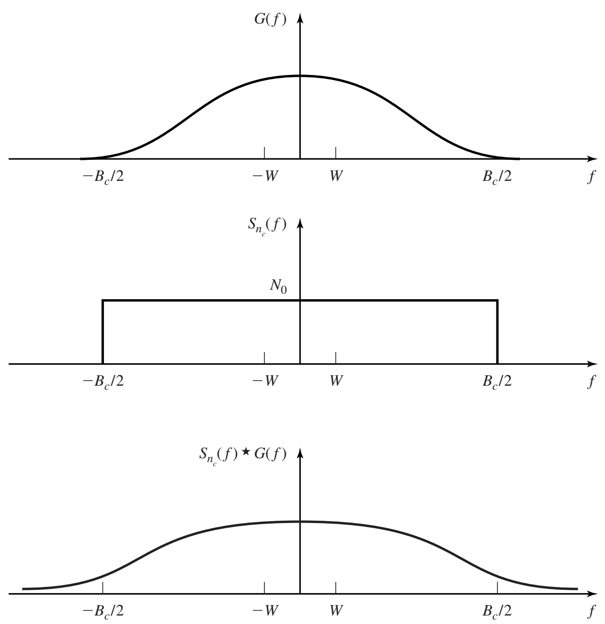


Figure 5.4: Typical plots of $G(f), S_{n_c}(f)$, and the result of their covolution

spectrum for $|f|<W$, the bandwidth of the message. From figure 5.4 it is obvious that for all

$|f| < W$, we have,

$$\begin{aligned}
S_Y(f) &= \frac{\exp -R_\Phi(0)}{{A_c}^2} S_{n_c}(f) \star G(f) \\
&= \frac{\exp -R_\Phi(0)}{{A_c}^2} N_0 \int_{\frac{-B_c}{2}}^{\frac{B_c}{2}} G(f) df \\
&\approx \frac{\exp -R_\Phi(0)}{{A_c}^2} N_0 \int_{-\infty}^{\infty} G(f) df \\
&= \frac{\exp -R_\Phi(0)}{{A_c}^2} N_0 g(\tau) \mid_{\tau=0} \\
&= \frac{\exp -R_\Phi(0)}{{A_c}^2} N_0 \exp R_\Phi(0) \\
&= \frac{N_0}{{A_c}^2}
\end{aligned} \tag{5.46}$$

It should be noted that equation 5.46 is a good approximation only for $|f| < W$. This means that for $|f| < W$, the spectrum of the noise components in the PM and FM case ar given by,

$$S_{n_0}(f) = \begin{cases} \frac{N_0}{{A_c}^2}, & PM \\ \frac{N_0}{{A_c}^2} f^2, & FM \end{cases} \tag{5.47}$$

where we have used the fact that in FM the noise component is given by $\frac{1}{2\pi}\frac{d}{dt}Y_n(t)$ as previously indicated. The power spectrum of the noise component at the output of the demodulator in the frequency interval $|f| < W$ for PM and FM is shown in Figure 5.5. It is interesting to note that PM

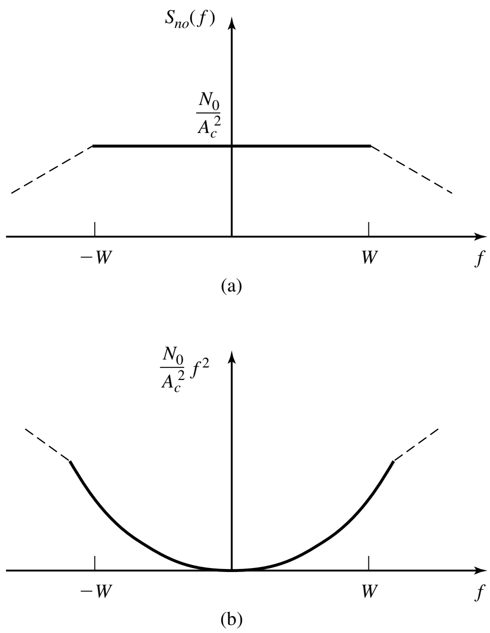


Figure 5.5: Noise power at demodulator output in (a) PM and (b) AM

has a flat noise spectrum and FM has a parabolic noise spectrum. Therefore, the effect of noise in FM for higher frequency components is much higher than the effect of noise on lower frequency components. The noise power at the output of the lowpass filter is the noise power in the frequency rang $[W,+W]$. Therefore, it is given by,

$$P_{n_0} = \int_{-W}^{+W} S_{n_0}(f)df = \begin{cases} \int_{-W}^{+W} \frac{N_0}{A_c{}^2}df, & PM \\ \int_{-W}^{+W} f^2 \frac{N_0}{A_c{}^2}df, & FM \end{cases} = \begin{cases} \frac{2WN_0}{A_c{}^2}, & PM \\ \frac{2W^3N_0}{3A_c{}^2}, & FM \end{cases} \tag{5.48}$$

Now we can use equation 5.36 to determine the output SNR in angle modulation. First, we have the output signal power,

$$P_{s_0} = \begin{cases} k_p{}^2 P_M, & PM \\ k_f{}^2 P_M, & FM \end{cases} \tag{5.49}$$

Then, the SNR, defined as

$$\left(\frac{S}{N}\right)_0 = \frac{P_{s_0}}{P_{n_0}} \tag{5.50}$$

becomes,

$$\left(\frac{S}{N}\right)_0 = \begin{cases} \dfrac{k_p{}^2 A_c{}^2}{2} \dfrac{P_M}{N_0 W}, & PM \\ \dfrac{3k_f{}^2 A_c{}^2}{2W^2} \dfrac{P_M}{N_0 W}, & FM \end{cases} \tag{5.51}$$

Noting that $\frac{A_c{}^2}{2}$ is the received signal power, denoted by P_R, and,

$$\begin{cases} \beta_p = k_p max|m(t)|, & PM \\ \beta_f = \frac{k_f max|m(t)|}{W}, & FM \end{cases} \tag{5.52}$$

we may express the output SNR as,

$$\left(\frac{S}{N}\right)_0 = \begin{cases} P_R \left(\dfrac{\beta_p}{max|m(t)|}\right)^2 \dfrac{P_M}{N_0 W}, & PM \\ 3P_R \left(\dfrac{\beta_f}{max|m(t)|}\right)^2 \dfrac{P_M}{N_0 W}, & FM \end{cases} \tag{5.53}$$

If we denote $\frac{P_M}{N_0 W}$ by $\left(\frac{S}{N}\right)_b$, the SNR of a baseband system with the same received power, we obtain,

$$\left(\frac{S}{N}\right)_0 = \begin{cases} P_M \left(\dfrac{\beta_p}{max|m(t)|}\right)^2 \left(\dfrac{S}{N}\right)_b, & PM \\ 3P_M \left(\dfrac{\beta_f}{max|m(t)|}\right)^2 \left(\dfrac{S}{N}\right)_b, & FM \end{cases} \tag{5.54}$$

Note that in the above expression $\frac{P_M}{max|m(t)|^2}$ is the average to peak power ratio of the message signal (or, equivalently, the power content of the normalized message, P_{M_n}). Therefore,

$$\left(\frac{S}{N}\right)_0 = \begin{cases} {\beta_p}^2 P_{M_n} \left(\frac{S}{N}\right)_b, & PM \\ 3{\beta_f}^2 P_{M_n} \left(\frac{S}{N}\right)_b, & FM \end{cases} \tag{5.55}$$

Now using Carson's rul $B_c = 2(\beta+1)W$, we can express the output SNR in terms of the bandwidth expansion factor, which is defined to be the ratio of the channel bandwidth to the message bandwidth and denoted by Ω.

$$\Omega = \frac{\beta_c}{W} = 2(\beta+1) \tag{5.56}$$

From this relationship we have $\beta = \frac{\Omega}{2} - 1$. Therefore,

$$\left(\frac{S}{N}\right)_0 = \begin{cases} P_M \left(\frac{\frac{\Omega}{2}-1}{max|m(t)|}\right)^2 \left(\frac{S}{N}\right)_b, & PM \\ 3P_M \left(\frac{\frac{\Omega}{2}-1}{max|m(t)|}\right)^2 \left(\frac{S}{N}\right)_b, & FM \end{cases} \tag{5.57}$$

5.2 Intermodulation

refers to the generation of unwanted frequencies resulting from the nonlinear mixing of two or more signals within an electronic device such as an amplifier, mixer, antenna, or connector. When signals of frequencies $f_1, f_2, \ldots$ pass through a nonlinear element, new frequency components called **intermodulation products** are generated according to:

$$f_{\text{IM}} = |mf_1 \pm nf_2|$$

where m and n are positive integers.

5.2.1 Origin

Intermodulation arises from the **nonlinearity** of active or passive components in the transmission chain. For instance, in an amplifier, the output voltage can be expressed by a Taylor series expansion:

$$y(t) = k_0 + k_1 x(t) + k_2 x^2(t) + k_3 x^3(t) + \cdots$$

The higher-order terms (especially $k_3 x^3(t)$) generate additional frequency combinations close to the desired signals, causing **interference and distortion**.

5.2.2 Order of Intermodulation Products

The **order** of an intermodulation product is defined by the sum $O = m + n$. Odd-order products, particularly the **third-order (IM3)** terms such as $2f_1 - f_2$ and $2f_2 - f_1$, are the most problematic because they fall within or near the useful frequency band and are difficult to filter.

5.2.3 Mitigation Methods

Several techniques can be used to reduce intermodulation effects:

- **Improving amplifier linearity** using predistortion, feedforward, or feedback linearization.
- **Applying selective filtering** at the transmitter output to suppress unwanted components.
- **Frequency planning** to avoid critical frequency combinations.
- **Increasing antenna isolation** and ensuring good mechanical quality of connectors and components to prevent passive intermodulation.
- **Reducing input power levels** to maintain operation within the linear region of amplifiers.

5.3 Conclusion

The study shows that linear modulation systems are more sensitive to noise than angle modulation systems, which provide better resistance due to their nonlinear nature. Understanding the effect of noise helps optimize the choice of modulation according to the application and transmission conditions, ensuring more reliable and higher quality communication.

6. Superheterodyne receiver

6.1 Introduction

The receiver's task is to extract the source information from the received modulated signal, which has been corrupted by noise. Often, the goal is for the receiver's output to be a replica of the modulating signal that was present at the transmitter's input. There are two main types of receivers: the tunable radio frequency (RF) receiver and the superheterodyne receiver. The RF receiver consists of several cascaded high-gain RF stages, with tunable bandpass filters set to the carrier frequency f_c, followed by an appropriate detection circuit (envelope detector, product detector, FM detector, etc.). The RF receiver is not widely used due to the difficulty in designing tunable RF stages that allow selecting the desired station while providing a narrow bandwidth capable of rejecting neighboring channel stations. Additionally, it is challenging to achieve high gain at radio frequencies while maintaining low parasitic coupling between the output and input of the RF amplification chain to prevent it from becoming an oscillator at f_c. Most receivers use the superheterodyne reception technique, as illustrated in Fig. 6.1. This technique involves converting, either by lowering or increasing the frequency of the input signal, to a practical frequency band called the intermediate frequency (IF). The information is then extracted using the appropriate detector. This basic receiver structure is used for receiving all types of bandpass signals, such as television, FM, AM, satellite signals, and radar signals. The RF amplifier has a bandpass characteristic that passes the desired signal and provides sufficient amplification to overcome the additional noise generated in the mixer stage. The RF filter characteristic also provides some rejection of signals and noise from adjacent channels, but the main rejection of adjacent channels is achieved by the IF filter.

The IF filter is a bandpass filter that selects either the up-conversion or down-conversion component (depending on the choice of the receiver designer). When up-conversion is selected, the complex envelope at the output of the IF filter (bandpass) is identical to the complex envelope at the RF input, except for the effect of RF filtering $H_1(f)$ and IF filtering $H_2(f)$. However, if down-conversion is used with $f_{LO} > f_c$, the complex envelope at the IF output will be the conjugate of the one at the RF input. This means that the sidebands of the IF output will be inverted (i.e., the upper sideband at the RF input will become the lower sideband at the IF output, and so on). If

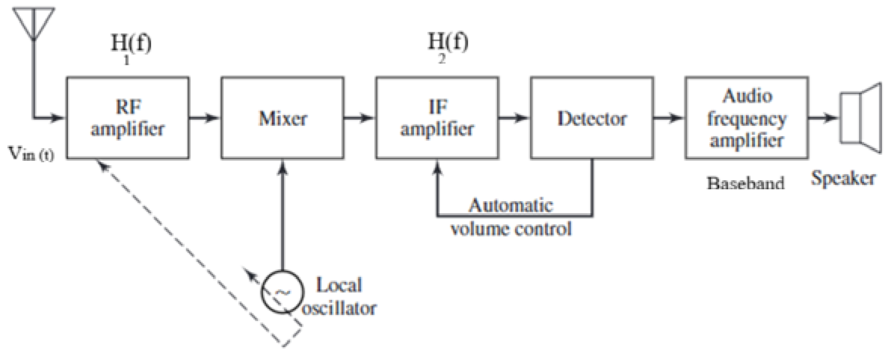


Figure 6.1: Superheterodyne AM receiver

$f_{LO} < f_c$, the sidebands are not inverted. In some specific applications, the local oscillator (LO) frequency is chosen to be equal to the carrier frequency $f_{LO} = f_c$, and the superheterodyne receiver then becomes a direct conversion receiver, also called a homodyne or synchrodyne receiver. In this case, the IF filter is replaced by a low-pass filter, so that the mixer-FPB combination becomes a product detector and the detection stage in Fig. 6.1 is removed. Thus, the direct conversion receiver is identical to an RF receiver with product detection.

The central frequency chosen for the IF amplifier is determined based on the following three considerations:

The frequency must allow for the use of a high-gain, stable amplifier. The frequency must be low enough so that, with practical circuit elements in the IF filters, Q values can be achieved, providing sharp attenuation outside the IF signal's passband. This reduces noise and minimizes interference from adjacent channels. The frequency must be high enough so that the receiver's image response can be kept acceptably low. The image response refers to the reception of an unwanted signal located at the image frequency, due to insufficient attenuation of this signal by the RF amplifier filter.

6.2 Automatic Volume or Gain Control (AVC)

The chief purposes of automatic volume control (AVC) or automatic gain control (AGC) in a radio receiver are to prevent fluctuations in loudspeaker volume when the audio signal at the antenna is fading in and out. An automatic volume control circuit regulates the radio frequency and intermediate frequency amplifiers in the receiver so that their gain is less for a strong signal than for a weak signal. In this way, when the signal strength at the antenna changes, the AVC circuit reduces the resultant change in the output voltage of the last intermediate frequency stage and consequently reduces the change in the speaker output volume. The AVC circuit reduces the radio frequency and intermediate frequency gain for a strong signal, usually by increasing the negative feedback to radio frequency, intermediate frequency, and frequency mixer stages when the signal increases. A simple AVC circuit is shown in Figure 6.2

On each positive half-cycle of the signal voltage, the diode conducts. Because of the flow of diode current through R_f, there is a voltage drop across R_1, which makes the point P negative with respect to ground. This voltage, applied through the filter R and C, acts as negative feedback to the preceding stages. When the signal strength at the antenna increases, therefore, the signal applied to the RF and IF amplifier stages is reduced due to the higher negative bias developed across the diode and resistor network. As a result, the overall gain of the receiver decreases, maintaining a nearly constant audio output level at the loudspeaker. Conversely, when the signal strength decreases, the bias voltage is reduced, allowing the gain of the RF and IF amplifiers to increase and thus

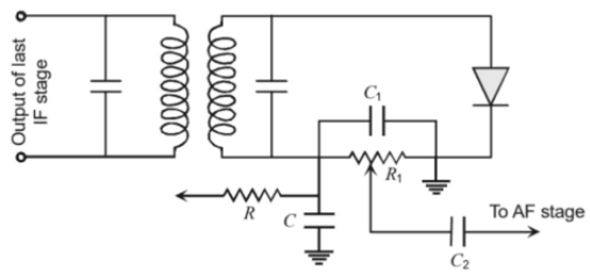


Figure 6.2: Automatic Volume control system

compensating for the weaker signal. This automatic adjustment helps maintain a uniform volume level and prevents distortion caused by overloading of the amplifier stages.

6.3 Automatic Frequency Control (AFC)

It is difficult to tune a superheterodyne receiver to a station accurately by merely listening to its output, because automatic volume control tends to maintain the receiver output constant. This difficulty is overcome by using the **Automatic Frequency Control (AFC)** circuit. AFC is used to tune FM receivers by adjusting the local oscillator frequency automatically. The AFC circuit, in conjunction with a discriminator (See Fig. 6. 3), maintains the receiver tuned to the desired frequency.

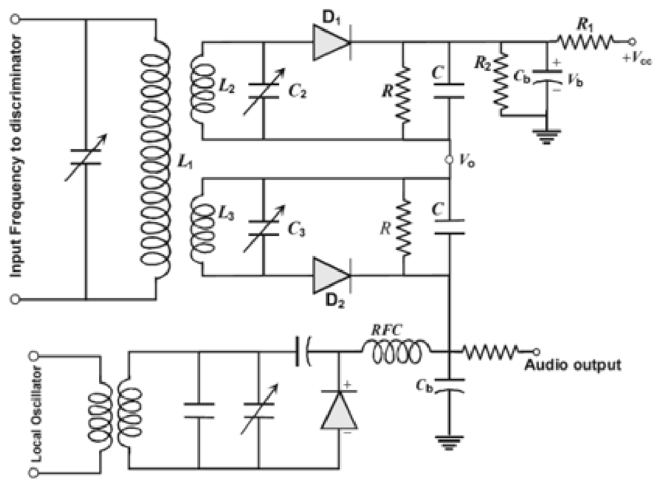


Figure 6.3: Automatic Frequency Control (AFC)

In the circuit, resistors R_1 and R_2 form a potential divider network that provides a bias voltage V_b. The sum of this bias voltage and the discriminator output voltage V_o provides the reverse bias to the varactor diode, expressed as:

$$V_r = V_b + V_o$$

where V_r is the reverse bias voltage applied to the varactor diode. When the receiver is correctly tuned to the desired station, the radio frequency signal produces an input frequency equal to the central frequency, and the output of the discriminator is zero. Thus, the reverse bias voltage of the varactor diode is $V_r = V_b$, and the oscillator is set at the input (center) frequency. If the signal frequency is high, the input frequency to the discriminator becomes lower than the central frequency. Hence, the output voltage of the discriminator V_o is positive. As a result, the reverse bias of the varactor diode increases ($V_r = V_b + V_o$), reducing its capacitance. Since the varactor diode is connected in parallel with the AFC tank circuit, this decrease in capacitance increases the frequency generated by the local oscillator, thereby increasing the input frequency and restoring it to the central value. Conversely, if the signal frequency is low, the input frequency to the discriminator becomes higher than the central frequency. In this case, the output voltage of the discriminator V_o is negative, which reduces the reverse bias on the varactor diode ($V_r = V_b + V_o$), causing its capacitance to increase. This increased capacitance lowers the oscillator frequency, thus decreasing the input frequency until it again matches the central frequency. Therefore, in both cases, the oscillator automatically adjusts itself to maintain the input frequency equal to the central (resonant) frequency. This automatic correction ensures stable reception and prevents frequency drift in FM receivers.

6.4 Superheterodyne Receiver for AM Broadcasting

Suppose a radio operating in the AM band is tuned to receive a station at 850 kHz, and the local oscillator (LO) frequency is higher than the carrier frequency. If the intermediate frequency (IF) is 455 kHz , then the LO frequency will be $850+455 = 1305$ (see Fig. 6.2). Furthermore, suppose other signals are present at the radio's RF input, particularly a signal at 1760 kHz. This signal will be down-converted by the mixer to $1760-1305 = 455$. In other words, the unwanted signal at 1760 kHz will be translated to 455 kHz and will be added to the mixer output alongside the desired signal at 850 kHz, which has also been converted to 455 kHz. This unwanted signal, which has been converted into the IF band, is called the image signal. If the RF amplifier gain is reduced, say by 25 dB at 1760 kHz compared to 850 kHz, and if the unwanted signal is 25 dB stronger at the receiver input than the desired signal, both signals will have the same level when translated into the IF. In this case, the unwanted signal will clearly interfere with the desired signal during the detection process. For down-converters: $f_{IF} = |f_c - f_{LO}|$. The image Frequencies:

$$f_{\text{image}} = \begin{cases} f_c + 2f_{\text{IF}}, & \text{if } f_{\text{LO}} > f_c \text{ (High Injection)} \\ f_c - 2f_{\text{IF}}, & \text{if } f_{\text{LO}} < f_c \text{ (Low Injection)} \end{cases} \tag{6.1}$$

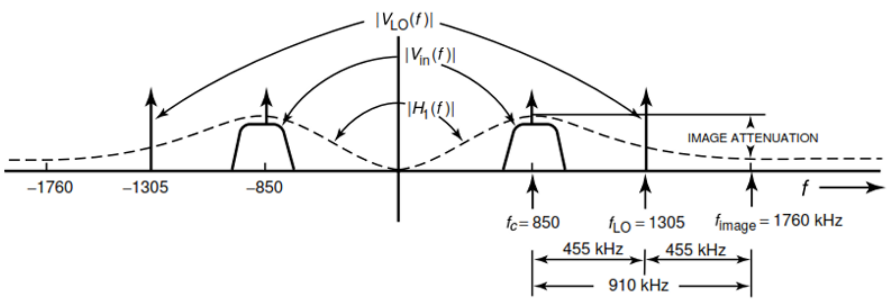


Figure 6.4: Signal spectra and transfer function of an RF amplifier in a superheterodyne receiver

Where f_c is the desired RF frequency, f_{IF} is the intermediate frequency (IF), and f_{LO} is the local oscillator frequency.

For up-converters (i.e., $f_{IF} = f_c + f_{LO}$), the image frequency is given by:

$$f_{\text{image}} = f_c + 2f_{LO}$$

From Fig. 6.1, it can be observed that the image response is generally reduced if the IF frequency is increased, since the image frequency f_{image} will be located farther from the main lobe (or peak) of the RF filter characteristic $|H(f)|$. It is also noted that other spurious responses (in addition to the image response) may appear in practical mixer circuits. These responses must also be taken into account in the design of a high-quality receiver. The type of detector chosen for a superheterodyne receiver depends on the intended application. For example, a product detector may be used in a digital system employing phase-shift keying (PSK) modulation, while an envelope detector is used in AM broadcast receivers. If the complex envelope $g(t)$ is required for generalized signal detection or for optimal reception in digital systems, the quadrature components $x(t)$ and $y(t)$, where $x(t) + jy(t) = g(t)$, can be obtained using quadrature product detectors, as illustrated in Fig. 6.3.

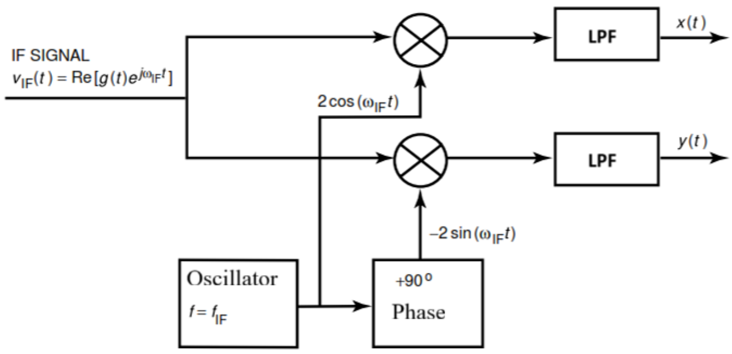


Figure 6.5: In-phase and quadrature (IQ) detector

In this case, $x(t)$ and $y(t)$ can be sent to a signal processor to extract the modulation information. By neglecting noise effects, the signal processor can recover $m(t)$ from $x(t)$ and $y(t)$, and consequently, demodulate the IF signal.

6.4.1 Advantages and disadvantages of the superheterodyne receiver

The superheterodyne receiver offers many advantages but also presents some drawbacks. Among its strengths are high gain and excellent stability, with the main advantage being the ability to achieve high gain without the risk of instability (self-oscillation). Parasitic coupling between the output and the input of the receiver does not cause oscillation, since the gain is distributed across distinct frequency bands: RF, IF, and baseband. Moreover, it is easy to tune the receiver to another frequency by adjusting the frequency of the local oscillator (LO) signal, which can be provided by a frequency synthesizer, and by changing the bandwidth of the RF amplifier to the desired frequency. High-quality factor (Q) elements, which are necessary to produce sharp filter slopes for adjacent channel rejection, are required only in the fixed-tuned IF amplifier. However, the superheterodyne receiver also has some disadvantages, particularly its sensitivity to spurious signals.

6.4.2 Interference

If the design is not carried out carefully, interference may occur. A discussion on receivers must also address the potential causes of interference. Often, the receiver owner believes that a particular signal such as an amateur radio transmission is responsible for the problem, but this is not always the case. The source of interference may originate from one of the following three sources:

1. **At the interference signal source:** The transmitter may generate out-of-band signal components (such as harmonics) that fall within the band of the desired signal.

2. **Within the receiver itself:** The input stage of the receiver (RF amplifier or mixer) may become saturated or produce spurious responses. Furthermore, cross-modulation can occur when the interfering signal drives the RF stage or mixer into a nonlinear operating region, resulting in distortion of the desired signal at the output of the RF amplifier.
3. **In the transmission channel:** Nonlinearity in the transmission medium may cause unwanted components to appear within the band of the desired signal.

6.5 Conclusion

In this chapter, we explored the principles and operation of the superheterodyne receiver used in AM broadcasting systems. The discussion highlighted its key advantage-high selectivity and sensitivity-which make it the most widely adopted architecture in radio communication. However, we also examined its main drawbacks, such as image frequency interference and circuit complexity. By understanding both the strengths and limitations of the superheterodyne design, engineers can better optimize receiver performance and develop improved architectures for modern communication systems.

7. Phase locked loop PLL

7.1 Introduction

The Phase-Locked Loop (PLL) is an essential feedback system widely used in electronics and communication engineering. Its primary function is to synchronize the frequency and phase of a controlled oscillator with those of an input reference signal. By continuously comparing the phase difference between the input and the output, the PLL adjusts the oscillator to maintain phase coherence.

This chapter introduces the fundamental operating principles of the PLL and defines its key performance parameters such as lock range, capture range, and pull-in time. These concepts are crucial for understanding the dynamic behavior and stability of the loop. In addition, practical characteristics and limitations of real-world PLLs are examined.

Finally, the chapter explores major applications of PLLs in modern electronic systems. Examples include synchronous demodulation, frequency synthesis, and signal recovery-functions that are essential in radio receivers, telecommunications, and digital communication circuits.

7.2 PLL Principles

The phase detector compares the input frequency f_s with the feedback frequency f_0 and generates an output signal which is a function of the difference between the phases of the two input signals. The output signal of the phase detector is a dc voltage. The output of phase detector is applied to low pass filter to remove high frequency noise from the dc voltage. The output of low pass filter, without high frequency noise is often referred to as error voltage or control voltage for VCO, is amplified an then applied as control voltage to VCO. When control voltage is zero, VCO is in free-running mode and its output frequency is called as center frequency f_0 as shown in Figure 7.1.

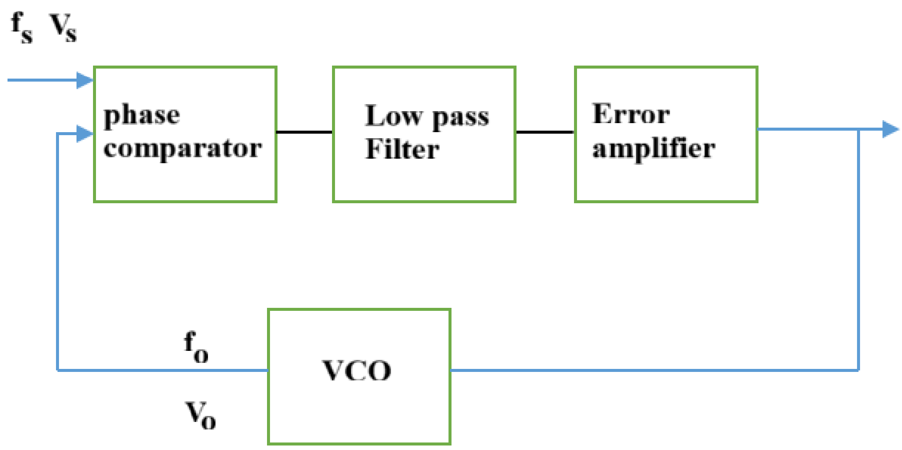


Figure 7.1: Block diagram of PLL

The non-zero control voltage results in a shift in the VCO frequency from its free-running frequency f_0 to a frequency f, given by $f = f_0 + K_v V_C$, where K_v is the voltage to frequency transfer coefficient of the VCO. The error or control voltage applied as an input to the VCO, forces the VCO to change its output frequency in the direction that reduces the difference between the input frequency and the output frequency of VCO. Once this action starts , we say that the signal is in the **capture range**. The VCO continues to change frequency till its output frequency is exactly the same as the input signal frequency. The circuit is then said to be locked. In locked condition, phase detector generates a constant dc level which is required to shift the output frequency of VCO from centre frequency to the input frequency. Once locked, PLL tracks the frequency changes of the input signal. Thus, a PLL goes through Three states: free running, capture and phase lock.

7.3 Definitions

7.3.1 Lock range

When PLL is in lock, it can track frequency changes in the incoming signal. The range of frequencies over which the PLL can maintain lock with the incoming signal is called the **lock range** or **tracking range** of the PLL. It is usually expressed as a percentage of f_0 the VCO frequency.

7.3.2 Capture range

The range of frequencies over which the PLL can acquire lock with an input signal is called the **capture range**. This parameter is also expressed as percentage of f_0.

7.3.3 Pull-in time

The total time taken by the PLL to establish lock is called **pull-in time**. This depends on the initial phase and frequency difference between the two signals as well as on the overall loop gain and loop filter characteristics.

7.3.4 Caracteristics

The *loop lock range* is represented as the range of frequencies about ω_0 for which the PLL maintains the relationship $\omega_i = \omega_{OSC}$ over a $\pm\frac{\pi}{2}$ range, then the lock range is defined as, $\omega_L = \pm\Delta\omega_{OSC}$. The capture range is the range of input frequencies within which an initially unlocked loop will get locked with an input signal. then the *capture range* is smaller than the *lock-range* as shown in Figure 7.3.

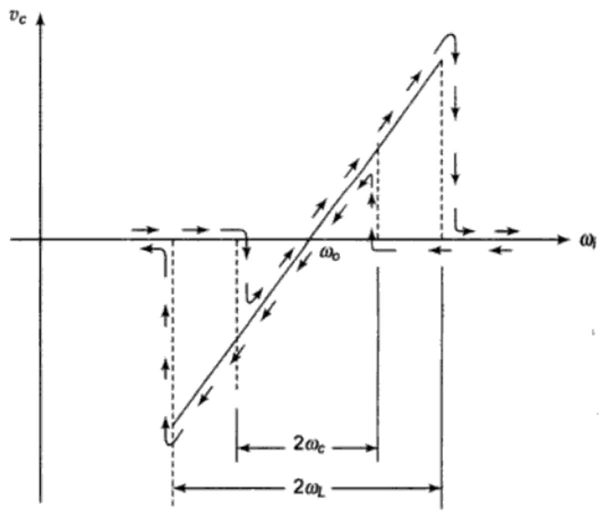


Figure 7.2: Locked and capture processes of PLL

7.4 7.4 Some Applications of the PLL

7.4.1 7.4.1 Principle of Synchronous Demodulation

Synchronous demodulation relies on a signal that must be perfectly synchronized with the carrier. This is where the phase-locked loop (PLL) comes into play. The PLL makes it possible to reconstruct the carrier from the modulated signal. From that point, the useful low-frequency modulating signal can be extracted, as shown in Figure 7.3.

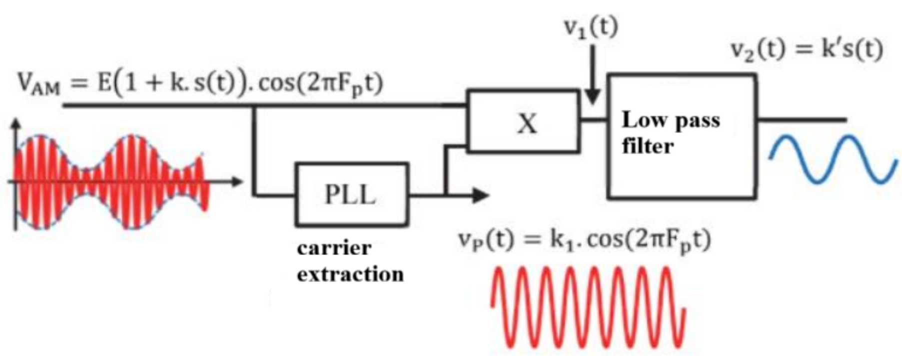


Figure 7.3: Demodulation using PLL

The PLL allows the extraction of the carrier from the amplitude-modulated signal. The amplitude-modulated signal and the signal representing the carrier, obtained at the output of the PLL, are simultaneously applied to a multiplier.

As a result, the output of the multiplier provides the following signal:

$$v_1(t) = E[1 + ks(t)]\cos(2\pi F_p t)\, k_1 \cos(2\pi F_p t) \tag{7.8}$$

$$v_1(t) = Ek_1[1 + ks(t)]\cos^2(2\pi F_p t) \tag{7.9}$$

$$v_1(t) = \frac{1}{2}Ek_1[1 + ks(t)][\cos(4\pi F_p t) + 1] \tag{7.10}$$

After filtering out the high-frequency components at $2F_p$ and the DC component, the output of the demodulation circuit yields a signal v_2 proportional to the original modulating signal $s(t)$:

$$v_2(t) = \frac{1}{2}Ekk_1 s(t) = k' s(t) \tag{7.11}$$

7.4.2 Frequency Synthesis

Frequency synthesis makes it possible to generate, from a reference signal, another signal whose frequency can vary over a wide range. The need to generate a signal with a frequency different from that of the reference signal is very important, for example, in telecommunications and signal transmission. The PLL is widely used to meet this requirement. In general, it is very difficult to design a variable-frequency oscillator with satisfactory stability. Frequency drift is always present due to various factors, the main one being temperature. The use of PLL-based servo systems with a stable reference (such as a quartz oscillator) makes it possible to solve these problems effectively. Frequency synthesis is derived from frequency multiplication, but the divider inserted between the VCO and the phase comparator is programmable according to the user's needs. For greater flexibility, another frequency divider can also be inserted between the reference signal and the phase comparator (see Figure 7. 4). The phase comparator receives, on its respective inputs, the frequency signals f_e/M and f_s/N. The numbers N and M are integers, and N is a programmable coefficient.

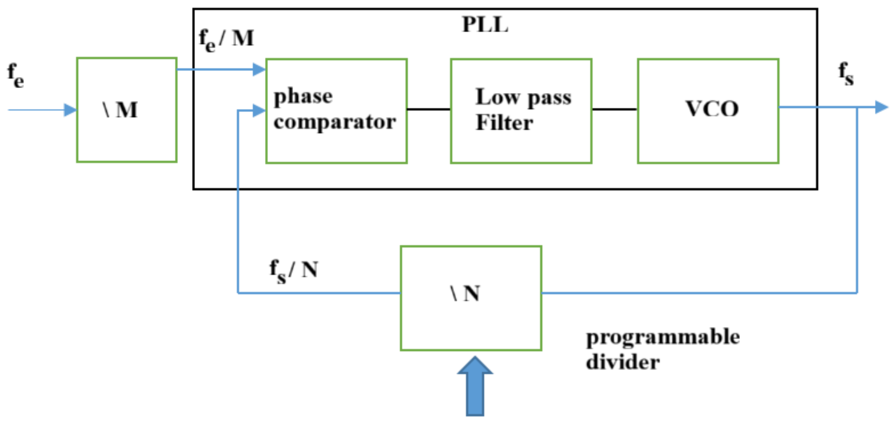


Figure 7.4: Frequency Synthesize

When the PLL is locked, we have:

$$\frac{f_e}{M} = \frac{f_s}{N} \tag{7.13}$$

Thus, the output frequency is given by:

$$f_s = N\frac{f_e}{M} \tag{7.14}$$

To demonstrate the usefulness of this application, suppose that we have a reference frequency of $f_e = 10$ MHz and we wish to produce all output frequencies ranging from 88 MHz to 108 MHz.

To achieve this, we simply set $M = 10$ and use a programmable coefficient N with step size 1, ranging from 88 to 108. This produces all output frequencies: 88 MHz, 89 MHz, 90 MHz, and so on.

7.5 Conclusion

The Phase Locked Loop is a highly versatile circuit that combines principles of feedback control, frequency comparison, and signal synchronization. Through its key parameters-lock range, capture range, and pull-in time it is possible to evaluate the PLL's ability to track and maintain synchronization with a reference signal under varying conditions. The study of PLL applications demonstrates its importance in both analog and digital systems. Whether used for coherent demodulation, clock generation, or frequency synthesis, the PLL provides high precision, stability, and flexibility. As technology continues to evolve, PLLs remain fundamental components in communication, navigation, and signal processing systems.

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