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Flow-Field Design and Analysis within Single and Dual-Bell Propulsion Nozzles

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ملخص: في هذه الدراسة، يتم دراسة تصميم وتحليل عددي لفهية دفع مزدوجة الانحناء، مع مقارنة أدائها وخصائص مجال السيلان الخاص بها بفوهة تقليدية ذات انحناء واحد تملك نفس نسبة مساحات المقاطع العرضية عند العنق والمخرج. لقد تم نمذجة الفوهة ذات الانحناء المزدوج، المعروفة بقدرتها على التكيف مع اختلاف الارتفاعات، بهدف استغلال نمطي تشغيلها: نمط على مستوى سطح البحر يتميز بانفصال السيلان، ونمط على ارتفاعات عالية يتميز بتماسك السيلان وارتباطه بجدران الفوهة. تم إجراء محاكاة في حالة الإستقرار باستخدام برنامج $\rightarrow$ ANSYS Fluent، مع حل معادلات $\rightarrow$ نافيه-ستوكس المعدلة حسب رينولدز (RANS) $\rightarrow$ في نموذج ثنائي الأبعاد متماثل المحور، باستعمال نموذج الاضطراب $k-\omega$ SST لإغلاق نظام المعادلات. تم تقييم أهم معالم السيلان مثل توزيع الضغط وتوزيع رقم $\rightarrow$ ماخ، إلى جانب الخصائص الأداء. وبالمقارنة مع الفوهة ذات الانحناء الواحد، تُظهر الفوهة ذات الانحناء المزدوج سلوكاً أكثر استقراراً للسيلان في ظروف فرط التمدد فضلاً عن كفاءة دفع أعلى تحت ظروف محيطية متغيرة. تُبرز هذه الدراسة المقارنة الإمكانيات الكبيرة التي توفرها الفوهات ذات الانحناء المزدوج في تحسين أداء مركبات الإطلاق من الجيل الجديد.

Résumé: La présente étude traite la conception et l'analyse numérique d'une tuyère de propulsion à double galbe, et compare ses performances ainsi que les caractéristiques de son champ d'écoulement à celles d'une tuyère classique à simple galbe ayant le même rapport de sections droites au col et à la sortie. La tuyère à double galbe, reconnue pour ses capacités d'adaptation en altitude, a été modélisée afin d'exploiter ses deux régimes de fonctionnement: un mode au niveau de la mer avec séparation de l'écoulement, et un mode en haute altitude avec attachement de l'écoulement aux parois. Des simulations en régime permanent ont été réalisées à l'aide du logiciel 'ANSYS Fluent', en résolvant les équations de Navier-Stokes moyennées selon Reynolds (RANS) en 2-D axisymétrique, avec le modèle de turbulence $k-\omega$ SST clôturant le système. Les paramètres d'écoulement les plus importants tels que la distribution de pression et la distribution du Mach, ainsi que les caractéristiques de performance ont été évalués. Comparée à la configuration à simple galbe, la tuyère à double galbe présente un comportement d'écoulement plus stable en conditions de sur-détente, ainsi qu'un meilleur rendement de poussée global sous des conditions ambiantes variables. Cette étude comparative met en évidence le potentiel des tuyères à double galbe pour l'amélioration des performances des premiers étages des lanceurs de nouvelle génération.

Abstract: The present study focuses on the design and numerical analysis of a dual-bell propulsion nozzle, and compares its performance and flow-field characteristics with those of a conventional single-bell nozzle having the same area ratio at the throat and exit. The

dual-bell nozzle, known for its altitude-adaptive capabilities, was modeled to exploit its two distinct operating modes: a sea-level mode with flow separation, and a high-altitude mode with flow attachment along the walls. Steady-state simulations were carried out using “ANSYS Fluent” platform that solves the Reynolds-Averaged Navier–Stokes (RANS) equations in 2-D axisymmetric form, with the $k-\omega$ SST turbulence model used to close the system. Key flow parameters such as pressure and Mach distributions along the performance characteristics, were evaluated. Compared to the single-bell configuration, the dual-bell nozzle exhibits more stable flow behavior under over-expanded conditions and delivers better overall thrust efficiency under varying ambient conditions. This comparative study highlights the potential of dual-bell nozzles to enhance the performance of the first stages of next-generation launch vehicles.

Key-words: Nozzle, double bell, single bell, supersonic, Ansys-Fluent, $k-\omega$ SST

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NOTATIONS

Coordinates system:

x : Axial coordinate
 y : Radial coordinate

Latin letters:

A_w : Coefficient of the polynomial simulating the nozzle profile
 A_e : Exit section
 A^* : Throat section
 a : Local speed of sound
 B_w : Coefficient of the polynomial simulating the nozzle profile
 C_f : Thrust coefficient
 C_w : Coefficient of the polynomial simulating the nozzle profile
 $C+$: Right-running characteristic
 $C-$: Left-running characteristic
 F : Thrust
 g : Gravitational acceleration
 I_{sp} : Specific impulse
 M : Mach number
 $\dot{m}$: Mass flow rate
 P : Static pressure
 P_0 : Total or stagnation pressure
 P^* : Static pressure at the speed of sound
 R_G : Ideal gas constant
 R_{tu} : Throat upstream radius of curvature
 R_{td} : Throat downstream radius of curvature
 T : Static temperature
 T_0 : Total or stagnation temperature
 t : Time
 u : Radial component of the velocity vector
 v : Axial component of the velocity vector
 V_{eff} : Effective velocity
 X_e : Length of the supersonic section
 Y_e : Radius of the exit cross-section

Greek letters :

α : Constant or Mach angle
 γ : Ratio of specific heats at constant pressure and volume
 δ : Coefficient

λ^+ : Right-running characteristic

λ^- : Left-running characteristic

θ : Angle

ρ : Density

Subscripts:

a : Ambient

e : Exit

j : junction or inflection point

tu : Throat upstream

td : Throat downstream

t : Total

θ : Stagnation

1 : First or base contour

2 : Second contour

Acronyms:

BL : Boundary Layer

CFD : Computational Fluid Dynamics

DBN : Dual-Bell Nozzle

NPR : Nozzle Pressure Ratio

SBN : Single Bell Nozzle

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